This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

TED
AN8
CUTTER

WENDT

Ger System
Uni of Wisconsin-Madison
728 State Street
Madison, WI 53706-1494
U.S.A.

TECHNICAL DRAWING SERIES

THE ESSENTIALS OF GEARING

A TEXT BOOK FOR TECHNICAL STUDENTS AND FOR SELF-INSTRUCTION,
CONTAINING NUMEROUS PROBLEMS AND
PRACTICAL FORMULAS

BY

GARDNER C. ANTHONY, A.M.

PROFESSOR OF DRAWING IN TUFTS COLLEGE; DEAN OF THE BROMFIELD-PEARSON SCHOOL;
AUTHOR OF "ELEMENTS OF MECHANICAL DRAWING," AND "MACHINE DRAWING,"
MEMBER OF AMERICAN SOCIETY FOR THE PROMOTION OF ENGINEERING EDUCATION;
MEMBER OF AMERICAN SOCIETY OF MECHANICAL ENGINEERS

BOSTON, U.S.A.
D. C. HEATH & CO., PUBLISHERS
1897

COPYRIGHT,
BY GARDNER C. ANTHONY,
1897.

44424
14 F '98
T'ED
.AN8
6937248

PREFACE.

THE most feasible method for the acquirement of a working knowledge of the theory of gear-teeth curves is by a graphic solution of problems relating thereto. But it requires much time on the part of an instructor, and is very difficult for the student, to devise suitable examples which, while fully illustrating the theory, shall involve the minimum amount of drawing. It is the aim of the author to overcome these difficulties by the presentation of a series of progressive problems, designed to illustrate the principles set forth in the text, and also to encourage a thorough investigation of the subject by suggesting lines of thought and study beyond the limits of this work.

In this as in the other books of the series the author would emphasize the fact that the plates are not intended for copies, but as illustrations. A definite lay-out for each problem is given, and the conditions for the same are clearly stated. This is accompanied by numerous references to the text, so that a careful study of the subject is necessitated before performing the problems.

Although specially addressed to students having no previous knowledge of the principles of kinematics, it is also designed to serve as supplementary to treatises on this subject.

The methods and problems have already proved their usefulness in the instruction of students of many grades; and it is hoped that their publication may promote a wider interest in, and more thorough study of, the essentials of gearing.

GARDNER C. ANTHONY.

TUFTS COLLEGE, *Sept.* 24, 1897.

CONTENTS.

CHAPTER I.

PAGE

Introduction. — General Principles 1

1. Constant Velocity Ratio. 2. Positive Rotation. 3. Gearing.

CHAPTER II.

Odontoidal Curves 4

4. Classes of Curves. 5. Cycloid. 6. Epicycloid. 7. Hypocycloid. 8. To Construct a Normal. 9. A Second Method for Describing the Cycloidal Curves. 10. Double Generation of the Epicycloid and Hypocycloid. 11. Epitrochoid. 12. Involute.

CHAPTER III.

Spur Gears and the Cycloidal System 8

13. Theory of Cycloidal Action. 14. Law of Tooth Contact. 15. Application. 16. Spur Gears. 17. Circular Pitch. 18. Diameter Pitch. 19. Face or Addendum. 20. Flank or Dedendum. 21. Path of Contact. 22. Arc of Contact. 23. Arcs of Approach and Recess. 24. Angle of Obliquity or Pressure. 25. Rack. 26. Spur Gears Having Action on Both Sides of the Pitch Point. 27. Clearance. 28. Curve of Least Clearance. 29. Backlash. 30. Conditions Governing the Practical Case. 31. Proportions of Standard Tooth. 32. Influence of the Diameter of the Rolling Circle on the Shape and Efficiency of Gear Teeth.

PAGE

33. Interchangeable Gears. 34. Practical Case of Cycloidal Gearing. 35. Face of Gear. 36. Comparison of Gears, illustrated in PLATES 4, 5, and 6. 37. Conventional Representation of Spur Gears.

CHAPTER IV.

Involute System 26

38. Theory of Involute Action. 39. Character of the Curve. 40. Involute Limiting Case. 41. Epicycloidal Extension of Involute Teeth. 42. Involute Practical Case. 43. Interference. 44. Influence of the Angle of Pressure. 45. Method for Determining the Least Angle of Pressure for a Given Number of Teeth Having no Interference. 46. Defects of a System of Involute Gearing. 47. Unsymmetrical Teeth.

CHAPTER V.

Annular Gearing 38

48. Cycloidal System of Annular Gearing. 49. Limiting Case. 50. Secondary Action in Annular Gearing. 51. Limitations of the Intermediate Describing Curve. 52. Limitations of Exterior and Interior Describing Curves. 53. The Limiting Values of the Exterior, Interior, and Intermediate Describing Circles for Secondary Action. 54. Practical Case. 55. Summary of Limitations and Practical Considerations. 56. Involute System of Annular Gearing.

CHAPTER VI.

Bevel Gearing 45

57. Theory of Bevel Gearing. 58. Character of Curves Employed in Bevel Gearing. 59. Tredgold Approximation. 60. Drafting the Bevel Gear. 61. Figuring the Bevel Gear with Axes at 90°. 62. Bevel Gear Table for Shafts at 90°. 63. Bevel Gears with Axes at Any Angle.

CHAPTER VII.

PAGE

Special Forms of Gears, Notation, Formulas, etc. 57

64. Odontographs and Odontograph Tables. **65.** Willis's Odontograph. **66.** The Three-Point Odontograph. **67.** The Grant Involute Odontograph. **68.** The Robinson Odontograph. **69.** The Klein Coördinate Odontograph. **70.** Special Forms of Odontoids, and Their Lines of Action. **71.** Conjugate Curves. **72.** Worm Gearing. **73.** Literature. **74.** Notation and Formulas.

CHAPTER VIII.

Problems 68

75. Method to be Observed in Performing the Problems —

PROBLEM 1. Cycloidal Limiting Case. Face or Flank Only 69
2. Cycloidal Limiting Case. Face and Flank 71
3. Cycloidal Gear. Practical Case 72
4. Involute Limiting Case 74
5. Involute Practical Cases 75
6. Cycloidal Annular Gear 76
7. Involute Annular Gear 77
8. Cycloidal and Involute Bevel Gears. Shafts at 90° 78
9. Cycloidal and Involute Bevel Gears. Shafts at Other than 90° 79

THE ESSENTIALS OF GEARING.

CHAPTER I.

INTRODUCTION.—GENERAL PRINCIPLES.

1. **Constant Velocity Ratio.** Motion may be transmitted between lines of shafting by means of friction surfaces; and if there be no slipping of the contact surfaces, the circumference of the one will have the same velocity as the circumference of the other. The number of revolutions of the shafts will be inversely proportional to the diameter of the friction surfaces, and this ratio will be maintained constant under the condition of no slip. Such friction surfaces and shafts are said to have a *constant velocity ratio.*

2. **Positive Rotation.** In order to transmit force, as well as motion, and to insure its being positive, it will be necessary to place cogs, or elevations, on one of the friction surfaces, and make suitable depressions in the other surface.

3. **Gearing.** The study of toothed gearing is a study of the shape of these cogs, teeth, or odontoids, which are designed to produce a *positive rotation* while preserving the condition of *constant velocity ratio.*

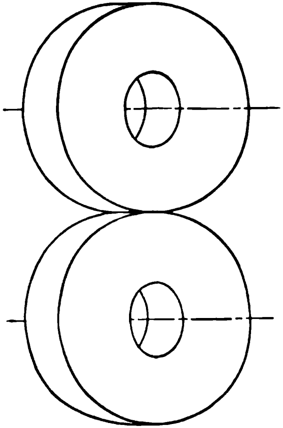


Fig. 1.

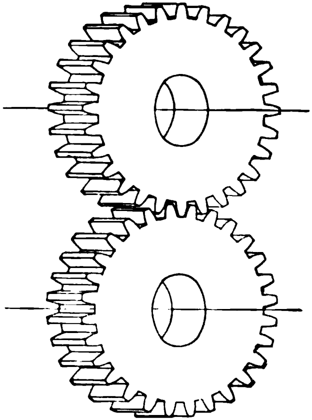


Fig. 2.

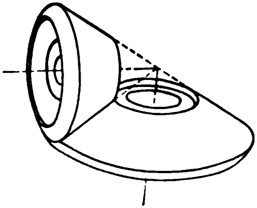


Fig. 3.

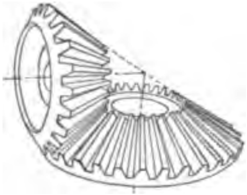


Fig. 4.

Gears may be classified as follows: —

1. If the shafts are parallel, the friction surfaces would be cylinders (Fig. 1), and the gears designed to produce the same condition, as to the velocity, are called *Spur Gears* (Fig. 2).

2. If the shafts intersect, the friction surfaces would be cones (Fig. 3), and the gears called *Bevel Gears* (Fig. 4).

3. If the shafts are neither intersecting nor parallel, the friction surfaces will be hyperboloids of revolution (Fig. 5), and the gears called *Hyperbolic, or Skew Gears* (Fig. 6).

In the preceding cases the elements of the teeth are rectilinear. and the friction surfaces touch each other along right lines.

4. If the elements of the teeth

in either of the first three cases be made helical, an entirely different class of gearing will result. The various forms are known as *Twisted*, *Spiral*, *Worm*, and *Screw Gearing* (Figs. 7 and 8). The action of the latter is analogous to that of a screw and nut.

One of these forms is generally employed as a substitute for hyperbolic, or skew gears, by reason of the difficulty experienced in correctly forming the teeth of such gears.

5. Another, although but little used, form, is that known as *Face Gearing*. The teeth are usually pins secured to the face of circular disks having axes perpendicular. The action takes place at a point only.

None of the latter forms can be represented by friction surfaces.

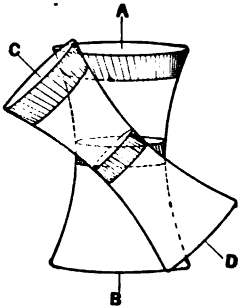


Fig. 5.

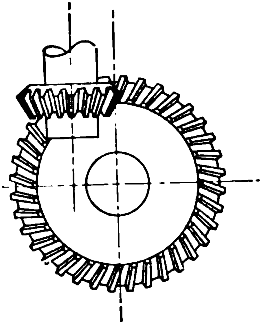


Fig. 6.

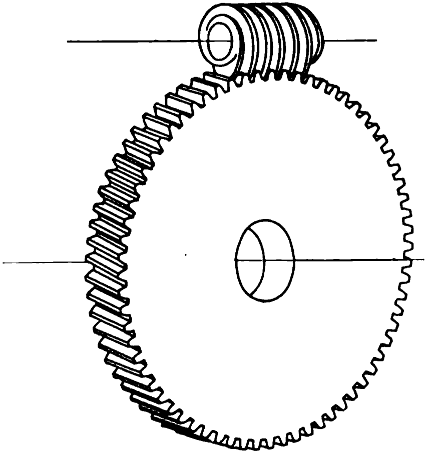


Fig. 7.

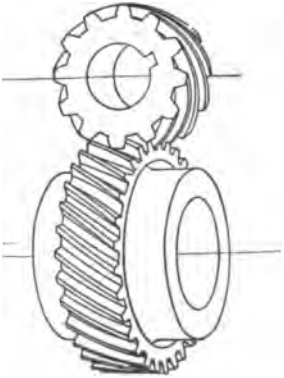


Fig. 8.

CHAPTER II.

ODONTOIDAL CURVES.

4. The two classes of curves commonly employed in gear teeth are the cycloidal and the involute. A knowledge of their characteristics and methods of generating is essential to an understanding of their application in gearing.

5. Cycloid. PLATE 1, Fig. 1. The cycloid is a curve generated by a point in the circumference of a circle which rolls upon its tangent. The circle is called the describing, or generating circle, and the point is known as the describing, or generating point. In Fig. 1, PLATE 1, B is the describing point, and B D C E the describing circle, which rolls on its tangent E B‴.

Assume a point, C, on the describing circle, and conceive the motion of the circle to be from left to right. As it rolls upon its tangent, the arc E C will be measured off on E B‴ until point C becomes a point of tangency at C′. The center of the describing circle will now lie at A′, in the perpendicular to E B‴ at C′.

From center A′, with radius of describing circle, draw the new position of describing circle. The generating point must lie in this circle at a distance from C′ equal to the chord B C.

Therefore, with radius equal to this chord, from center C′, describe an arc intersecting the new position of the describing circle. The line B′ C′ is called the instantaneous radius, or *normal*, of the curve at B′, it being a perpendicular to the tangent of the curve at this point.

The normal at B″ would be B″ D′. The radius A′ B′ is known as the *describing*, or *generating radius*, and A′ C′ is the *contact radius*, or the radius at the point of contact. In like manner other positions of the describing point may be found, and the curve connecting them will be the cycloid required.

6. Epicycloid. PLATE 1, Fig. 2. If the describing circle rolls upon the outside of an arc, or circle, called the director circle, the curve generated will be an epicycloid, Fig. 2, PLATE 1. The method of describing this curve is similar to that for the cycloid, and the lettering is the same. It must be observed, however, that any contact radius, as A′ C′, is a radial line of the circle on which it rolls.

7. Hypocycloid. PLATE 1, Fig. 3. If the describing circle rolls upon the inside of a circle, the curve generated will be an hypocycloid. Fig. 3 illustrates this curve, the same lettering being used as that of the preceding cases.

If it be required to draw a normal at any point of this, or the two preceding curves, the following method may be employed: —

8. To Construct a Normal. From the given point on the curve, as a center, with radius of generating circle, describe an arc cutting the path described by the center of the generating circle. From this point draw the contact radius, thus obtaining the contact point. Connect this with the given point, and the line will be the required normal.

9. A Second Method for Describing the Cycloidal Curves. PLATE 2, Fig. 1. A B C is a director circle, A D E, the generating circle for the epicycloid A A′ A″ H, and A K L the generating circle for the hypocycloid A L C.

To describe the epicycloid, assume any point, D, on the generating circle, and lay off the arc A D′ on the director circle, making it equal to arc A D. If A be the describing point, then A D will be the normal when D shall have become a contact point, as at D′. With L as a center, describe the arc D A′. The describing point A must be in this arc when D shall be at D′. From D′ as a center, with radius equal to the chord A D, describe an arc intersecting A′ D, and thus determine A′, a point in the epicycloid. Similarly obtain other points, and draw the curve.

The hypocycloid may be constructed in like manner, as shown by the same figure. This also illustrates a special case in which the hypocycloid is a radial line, A L C, and this is due to the diameter of the describing circle being equal to the radius of the director circle.

The same method may also be employed in the construction of the cycloid.

10. Double Generation of the Epicycloid and Hypocycloid. PLATE 2, Fig. 1. The epicycloid may always be generated by either of two describing circles, which differ in diameter by an amount equal to the diameter of the director circle. Thus in the case illustrated, the epicycloid A A′ A″ H may be generated by the circle A D E, with A as a describing point, or by the circle S T H, with H as a describing point. Similarly the hypocycloid is capable of being generated by either of two rolling circles, the sum of which diameters must equal that of the director circle.*

11. Epitrochoid. PLATE 2, Fig. 2. When the describing point does not lie on the circumference of the generating circle, a curve, commonly called an epitrochoid, is described. If the point lies without the circle, as at B, a looped curve, B B′ B″, called the curtate epitrochoid, is described; and if the point be within, as at D, the curve will be a prolate epitrochoid, as D D′ D″.

---

* For the geometrical demonstration of this problem see the appendix of Professor MacCord's "Kinematics," page 319.

To obtain a point in the former, assume any point, C, in the circumference of the describing circle, and determine its position, C′, when it shall have become a contact point. Draw the contact radius A′ C′, and from C′ and A′ as centers, with radii A B and C B, describe arcs intersecting at B′, a point in the curve. B′ C′ is the normal at this point. In like manner obtain the point D′ in the prolate epitrochoid.

12. Involute. PLATE 1, Fig. 4. The involute is a curve generated by a point of a tangent right line rolling upon a circle, known as the *base circle*, or the describing point may be regarded as at the extremity of a fine wire which is unwound from a cylinder corresponding to the base circle. In Fig. 4, A B C D is the arc of a base circle, and A the point from which the involute is generated.

Lay off arcs A B, B C, C D, preferably equal to each other, and from points B, C, and D, draw tangents equal in length to the arcs A B, A C, and A D. A line drawn through the extremity of these tangents will be an involute of the base circle A B C D.

CHAPTER III.

SPUR GEARS AND THE CYCLOIDAL SYSTEM.

13. **Theory of Cycloidal Action.** PLATE 3, Fig. 1. Let H K and M L be the peripheries of two disks, having centers G and F, and S the center of a third disk, also revolving in contact with the arcs H K and M L. The largest disk will be known as disk 1, the second size as disk 2, and the smallest as disk 3, or the describing disk. Consider the peripheries of these disks in contact at A, so that motion imparted to one will produce an equal motion in the circumference of the other two, thus maintaining at all times an equal circumferential velocity, or constant velocity ratio.

Imagine this to represent a model, disk 1 having a flange I O extending below the other disks, and the describing disk as being provided with a marking point at A, each of the disks being free to revolve about their respective axes. Consider first the relation between the describing disk and disk 1, the marking point being at A. Suppose motion to be given disk 3 in the direction indicated by the arrow, so that the describing point shall move from A to A′. The point C of disk 1, which coincides with A when the describing disk is in the first position, will now have moved on the circumference H K, to C′, an arc equal to A A′. During this time, the curve A′ C′ will have been drawn upon the flange of disk 1 by the marking point. Next, suppose the marking point to move from A′ to A″, then, since the circumferences of these disks traverse equal spaces in equal times, C′ will have revolved to C″, and the curve A′ C′ will now occupy the position E″ C″. But, since the marking point has continued to describe a curve upon the flange of disk 1, the curve E″ C″ will be extended to A″. In like manner the marking point moves to A‴, continuing to describe a curve, as C″ A″ revolves to C‴ A‴. If now the describing disk be freed from the axis on which we have supposed it to revolve, and be rolled on the circumference H K, the marking point would describe the same curve, A‴ E‴ C‴, as that already drawn, which is an epicycloid.

In the same manner, we may imagine the marking point to describe a curve upon disk 2, which curve, in its successive positions, would be shown by A′ B′, A″ B″, and A‴ B‴. For the same reason, too, the arc A A′ A″ A‴ will equal the arc B B′ B″ B‴; and if, in a manner similar to the preceding, we roll the describing disk on the inside of the arc M L, we shall describe the same curve A‴ D‴ B‴, and find it to be an hypocycloid.

Again, consider these curves, A‴ C‴ and A‴ B‴, as being traced at the same time by the describing point A. If we now observe any special position of the point, as A″, it will be seen to be common to an epicycloidal, and a hypocycloidal curve, which have a common normal, A″ A, intersecting the line drawn through the centers, F and G, at the point of tangency of the disks. This condition is true for all positions of the two curves.

If these curves A‴ C‴, and A‴ B‴, be now used as the outlines for gear teeth, as in Fig. 2, G and F being the centers and H K and M L the pitch lines, we shall have obtained a positive rotation with a uniform velocity ratio, for it was under this latter condition that the curves were generated, and the common normal to the curves at any point of contact will pass through the point A (the pitch point). Such curves are said to be conjugate.

It is not necessary that the describing point be on the circumference of the circle, or that the describing curve be a circle, in order to obtain two curves which, acting together, shall produce a constant velocity ratio.

14. Law of Tooth Contact. In order to preserve the condition of constant velocity ratio, the tooth outlines which act in contact must be such that the common normals at the point of contact shall always cut the line of centers in the same point; and in general, the curves must be such as may be simultaneously traced upon the planes of rotation of two disks, while revolving, by a marking point which is carried by a describing curve, moving in rolling contact with both disks.

15. Application. Suppose action to take place between the odontoids, or gear teeth, shown in Fig. 2, PLATE 3. Let 1 be the driver, and suppose motion to begin from the position shown in the figure, the contact being at A. As the motion takes place, points A′, A″, A‴, will successively come into contact, their common normals passing through the PITCH POINT, A, at the time of their contact, thus producing a constant velocity ratio, and the periphery, or PITCH LINE, of 1, will have the same velocity as the periphery, or pitch line, of 2. But this uniform motion must cease when points A‴ A‴ come into contact, and the velocity ratio will remain constant no longer, unless a second pair of curves begin contact at this moment.

PLATE 4 illustrates a pair of disks provided with a series of these curves arranged so as to continue the motion indefinitely in either direction.

16. Spur Gears. PLATE 4. F is the center of a pinion having twelve teeth, and G the center of a gear of eighteen teeth, only a segment of the latter being shown. A C K is the describing circle, carrying the marking point C, which described the epicycloid C D, and the hypocycloid C E. The depth of the pinion tooth must be made sufficient to admit the addendum of the gear tooth, but only that portion of the curve between C and E will engage

C D. The remainder of the pinion flank may be a continuation of the hypocycloid, or any other curve which may not interfere with the action of the gear tooth. The opposite sides of the teeth are made alike in order that motion may take place in either direction. If the direction be that indicated by the arrows, the pinion being the driver, the shaded side of the teeth would have contact; and if the direction be reversed the opposite faces would engage.

In order to accurately reproduce the dedenda of the pinion, a scroll may be used in the following manner: —

Having selected one to match the tooth curve, C E, continue the curve of the scroll by the center F, from which a circle should be drawn tangent to the line of the scroll. Mark that point of the scroll in contact with the pitch circle. Having laid off the pitch, and thickness of the teeth, place the marked point of the scroll to coincide with these points, and at the same time tangent to the circle already drawn. Draw such part of the curve as lies between the addendum and dedendum circles. Reverse the scroll for drawing the opposite side of the teeth.

17. Circular Pitch. The distance A D, or A E, measured on the pitch line between corresponding points of consecutive teeth, is called the circular pitch, and is equal to the $\frac{\text{circumference of pitch circle}}{\text{number of teeth}}$.

Let P' denote the circular pitch, D' the diameter of the pitch circle, and N the number of teeth, then will $P' = \frac{\pi D'}{N}$ (1), and, $\frac{N}{D'} = \frac{\pi}{P'}$ (2).

18. Diameter Pitch. In order to express in a more direct and simple manner the ratio between the diameter of the pitch circle and the number of teeth, and to easily determine the proportions of the teeth, it has been found expedient to apply the term pitch, or more properly, diameter pitch, to designate the ratio between the number of teeth and the diameter of pitch circle. ***This is not an absolute measure, but a ratio;*** and since it may usually be expressed by a whole number, the proportions of the parts of a tooth, which are commonly dependent on the pitch, may be more readily determined, and all the figuring of the gear simplified.

Designating the diameter pitch by P, $P = \frac{N}{D'}$ (3).

To obtain the relation between the diameter pitch and the circular pitch, compare formulas 2 and 3. $\frac{N}{D'} = \frac{\pi}{P'}$, $\frac{N}{D'} = P$; hence $\frac{\pi}{P'} = P$ or $P\,P' = \pi$ (4). This last equation expresses the relation between the two pitches in a simple form which may be easily remembered.

ILLUSTRATION. — The pinion represented in PLATE 4 has 12 teeth, and is 3 inches in diameter. $\frac{N}{D'} = P$, $\frac{12}{3} = 4$. The pitch, therefore, is 4. The circular pitch, $P' = \frac{\pi}{P} = \frac{3.1416}{4} = .7854$. Having given any two of the terms, N, D′, P, P′, the other terms may be determined.

19. Face, or Addendum. That portion of the tooth curve lying outside of the pitch circle is called the face or addendum, as C D, PLATE 4.

20. Flank, or Dedendum. That portion of the tooth curve lying inside of the pitch circle is called the flank or dedendum, as E H, PLATE 4.

21. Path of Contact. In Fig. 1, PLATE 3, it will be observed that the contact between the two curves takes place in the arc A A′ A″ A‴. This is called the path of contact, or line of action, and in the cycloidal system this line is an arc of the describing circle.

22. Arc of Contact. The arc described by a point on the pitch line during the time of contact of two odontoids is called the arc of contact. It must not be less than the pitch. In this case the arc of contact would be measured by the arcs A D or A E, and these arcs being equal to the pitch, the case is called a limiting one. In practice it should be greater, which would be accomplished by lengthening the addendum.

23. Arcs of Approach and Recess. There are four cases of contact that may take place between the gear and pinion of PLATE 4.

1. Gear as driver. Direction opposed to the arrows. Contact begins at A and ends at C.
2. Pinion as driver. Direction same as arrows. Contact begins at C and ends at A.

In each of these cases the action will take place between the shaded portions of the teeth.

3. Gear as driver. Direction same as arrows. Contact begins at A and ends at L.
4. Pinion as driver. Direction opposed to the arrows. Contact begins at L and ends at A.

In the last two cases there will be no contact between the shaded portions of the teeth.

In the first and third cases the contact takes place from the pitch point, and is called an arc of recess.

In the second and fourth cases the contact takes place toward the pitch point, ending at A, and is called an arc of approach.

It should also be observed that in the case illustrated the arc of contact must be either one of approach or of recess; but had the teeth of each gear been provided with curves on both sides of the pitch-line, as in PLATE 5, the arc of contact would have consisted of an arc of approach and of recess. (See ART. 30, page 16, for a further discussion of the relation between these arcs.)

24. Angle of Obliquity, or Pressure. The angle which the common normal to a pair of conjugate teeth makes with the tangent at the pitch point, is called the angle of obliquity, or angle of pressure. The angle C A P, PLATE 4, is the angle of greatest obliquity. The greater this angle, the greater the tendency to thrust the gears apart; the friction will be increased and the component of force tending to produce rotation will be decreased.

25. Rack. If the diameter of the gear be indefinitely increased, the pitch circle will finally become a right line, and the gear will then be known as a rack.

The rack shown in PLATE 4 has teeth only on one side of the pitch line, like the pinion and gear, and the conditions of action are similar. The tooth-curve will be a cycloid, and the rolling circle, M N O, must be the same as that used for the engaging pinion, in order to fulfil the general law for maintaining a constant velocity ratio (ART. 14, page 10).

26. Spur Gears having action on both sides of the Pitch Point, PLATE 5. If we assume the diameters of pitch and rolling circles to be the same as before, and the arc of action, C A, unchanged, the addendum of gear and dedendum of pinion will be the same as those of PLATE 4. This case, however, differs from the preceding in that the number of teeth is but half as great, and therefore the pitch will be doubled. This will require the arc of action to be doubled, in order that it shall equal the pitch (ART. 22, page 13). Such increase in the arc of action may be made by continuing the path of contact to the other side of the pitch point, following the circumference of a rolling circle which may or may not be equal to the other rolling circle. Having laid off the arc A H equal to one-half the circular pitch, describe the curves H K and H L, with H as the generating point of the new rolling circle. The former of these curves will become the addendum of the pinion, and the latter the dedendum of the gear tooth. The engaging gears will then have both faces and flanks, the action will begin at C and end at H, the path of contact will be C A H, the arc C A being the path of approach, and A H the path of recess, their sum being equal to the circular pitch.

In a similar manner the dedendum of the rack tooth may be described to engage the addendum of the pinion tooth, and the contact begun at N will end at O, N M being the path of approach, and M O the path of recess. That portion of the dedendum of rack tooth which engages the addendum of the pinion is indicated by sectioning, but it is necessary to continue the dedendum to a depth sufficient to allow the addendum of the engaging tooth to enter.

27. Clearance. The space between the addendum circle of one gear and the dedendum circle of an engaging gear is called clearance. Fig. 9, page 17.

28. Curve of Least Clearance. If the pitch circle of the gear be rolled on that of the pinion, and the epitrochoid of the highest point, C, of the gear tooth be determined, it will be the curve of least clearance.

The successive positions of the tooth, when so revolved, are shown by the dotted line in PLATE 5, and the line connecting these points would be the desired curve. This may be obtained as follows: Assume any point, R on the pitch circle of pinion, and lay off arc A R′ on the pitch circle of gear, equal to arc A R. From R, with radius R C′, equal to R′ C, describe an arc. Similarly describe other arcs, and draw a curve touching these arcs on the inside. This curve will be the curve of least clearance.*

---

* See also the method of ART. 71, page 61.

29. Backlash. In order to allow for unavoidable inaccuracies of workmanship and operating, it is customary to make the sum of the thickness of two conjugate teeth something less than the circular pitch. This insures contact between the engaging faces only.

30. Conditions governing the practical case. From a consideration of the foregoing limiting cases, the following principles are deduced, to which are also added the limitations and modifications established by practice.

1. The curves of gear teeth, which act to produce a constant velocity ratio, must be described by the same circle rolling in contact with their respective pitch circles. (ART. 14, page 10.) Practical considerations limit the diameter of the describing circle to a maximum of about $\frac{D'}{2}$, or equal to the radius of the pitch circle, and a minimum of about $1\frac{3}{4}$ P′, or 5.5 P. See also ART. 32, page 19.

2. The arc of contact must equal the circular pitch, and in practice exceed it as much as possible.

3. The addendum of a gear tooth engages the dedendum of the pinion, and the action between them either begins or ceases at the pitch point.

Since the addendum and dedendum of any tooth are independent curves, they may be described by rolling circles differing in diameter.

4. In the limiting cases considered, the height of the tooth is dependent on the arc of contact, but in practice, the arc of contact is made dependent on the height of the tooth.

While it is an almost universal custom to make the addenda of engaging teeth equal, there are special cases, in which very smooth-running gears are required, where it would be advantageous to make the addenda of the driver less than those of the driven gear, thus increasing the arc of recess, or decreasing the arc of approach.

P = Diam. Pitch
P' = Circ. Pitch

The approaching action being more detrimental, by reason of the friction induced, it is common to design clock gears so as to eliminate this by providing the driver with faces only, and the driven with flanks only. Or, if the gears are made with both faces and flanks, to so round the faces of the driven gear that no action may take place.

31. Proportions of Standard Tooth. The proportions most commonly accepted for cut gears are those illustrated in Fig. 9. The dimensions are made dependent on the pitch, as follows: —

ADDENDUM, $(S) = \frac{1}{\text{diam.-pitch}} = \frac{1}{P}$.

DEDENDUM, $(s + f) = \frac{1}{\text{diam pitch}} + \text{clearance} = \frac{1}{P} + f$.

THICKNESS, $(t) = \frac{1}{2}\text{ circular pitch} = \frac{P'}{2} = \frac{\pi}{2P}$.

CLEARANCE, $(f) = \frac{1}{8}\text{ addendum} = \frac{S}{8} = \frac{1}{8P}$ or, $f = \frac{1}{10}$ thickness of tooth $= \frac{t}{10} = \frac{P'}{20} = \frac{\pi}{20P}$.

In assuming this value for the thickness of the tooth the backlash is taken as zero, but of course the

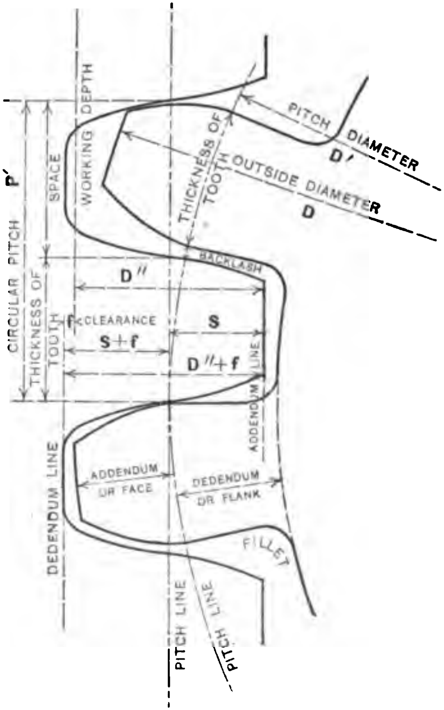


Fig. 9.

tooth must be slightly smaller than the space to permit of freedom in action. If there be any backlash the value of t will be $\frac{\text{circular pitch} - \text{backlash}}{2}$. In rough cast gears the backlash may be as great as $\frac{1}{12}$th the circular pitch, but this amount is very excessive. It is, however, inconsistent to base the values for backlash, or clearance, on the pitch, since an increase in the size of the tooth, or pitch, does not necessarily mean a proportional increase in the allowance to be made for the inaccuracies of workmanship. Indeed, both these clearances must be left to the judgment of the designer.

FILLETS. The circular arc tangent to the flank and dedendum circle is called the fillet. It is designed to strengthen the tooth by avoiding the sharp corner at the root of the tooth. A good rule is that of making the radius of fillet equal to one-sixth of the space between the teeth, measured on the addendum circle, as in Fig. ~~6~~.9 The limit of size may be determined by obtaining the curve of last clearance. ART. 28, page 15.

32. Influence of the Diameter of the Rolling Circle on the Shape and Efficiency of Gear Teeth. If the height of the teeth be previously determined, any increase in the diameter of the describing circle will increase the path of contact and decrease the angle of pressure. But since an increase in the diameter of the describing circle produces a weaker tooth, by reason of the undercutting of the flank, as shown in Fig. 12, page 21, the maximum limit of the diameter is commonly made equal to the radius of the pitch circle within which it rolls. As was shown in ART. 9, page 5, this will generate a radial flank. In the case of gears designed to transmit a uniform force, and not subjected to sudden shocks, it is desirable that the teeth have radial flanks, and consequently the diameters of the rolling circles will be equal to the radii of the pitch circles within which they roll. If the force to be transmitted be irregular, and the teeth required to sustain sudden strains, it is better that the flank be made wider at the dedendum circle, and a describing circle chosen of a diameter sufficiently small to produce the desired result.

In general, the diameters of the describing circles will lie between the values of $\frac{D'}{2}$ and $\frac{5}{P}$. The second value was used for the describing circle of the gears in PLATE 5, and would describe radial flanks for a gear having ten teeth.

Fig. 10 illustrates the effect of a change in the diameter of the rolling circle on the path of contact and angle of pressure. Two gears of equal diameter are supposed to engage, and the teeth are described by rolling circles of equal diameter.

K P is the addendum, and P L the dedendum of the tooth described by the rolling circles C P, and P D, which are of the same diameter, and equal to one-quarter of the pitch diameter. A C being the addendum line of the engaging gear, C may be considered as the first, and D as the last, point of contact. The arcs C P and P D constitute the path of contact, and the angle C P H is the angle of pressure.

Next consider the describing circle as increased, its

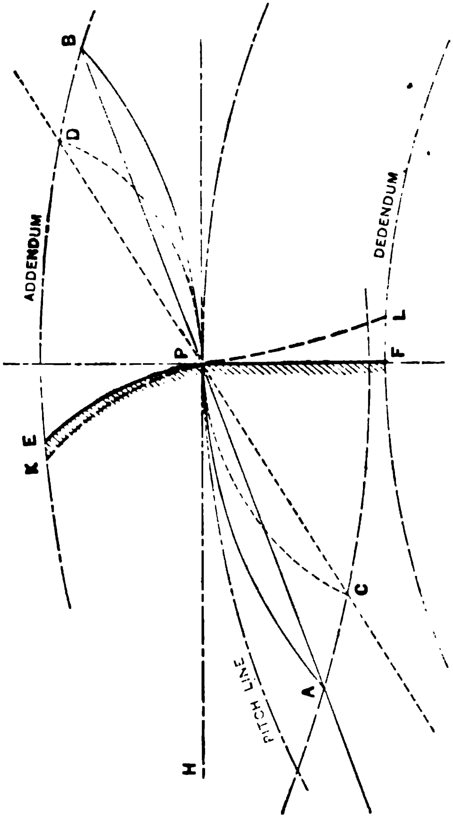


Fig. 10.

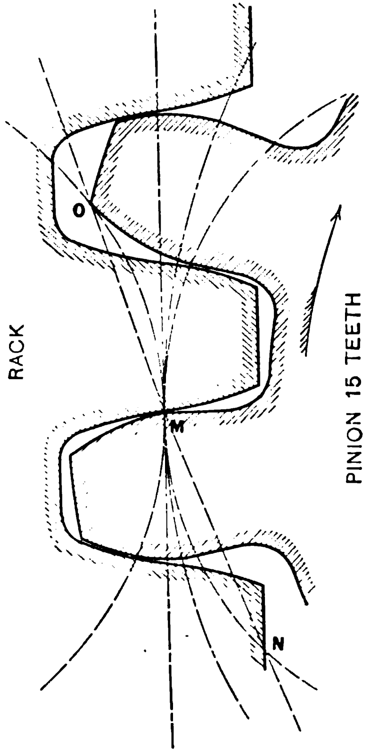


Fig. 11.

diameter being equal to one-half of the diameter of the pitch circle. The form of the tooth will now be E P F, and the path of contact A P B. In the latter case the arc of contact will be greater, the maximum angle of pressure less, and the tooth weaker than in the former.

The relation between the two cases may be more exactly stated as follows: —

Diameter of describing curve,	$\frac{D'}{4}$	$\frac{D'}{2}$.
Arc of contact,	1.08 P′	1.35 P′.
Maximum angle of pressure,	32.3°	20.3°.

Again, the weakness of the tooth in the second case may be partially overcome by reducing the height of the tooth, and in general this would be advantageous, the so-called standard tooth being too high for the best results.

33. Interchangeable Gears. Since the same diameter of rolling circle must be used for the addendum of pinion tooth, and the dedendum of engaging gear tooth, it follows that for any system of interchangeable gears, the addenda and dedenda of all teeth must be described by the same describing curve. It is also necessary that the pitch, and proportion of the teeth, be constant.

In practice, it is common to regard gears of twelve or of fifteen teeth as the base of the system, and the diameter of the rolling circle is made equal to the radius of the corresponding pitch circle, thus describing teeth with radial flanks for the smallest gear of the set. If twelve be adopted as the smallest number of teeth in the system, the diameter of the pitch circle will be $D' = \frac{N}{P} = \frac{12}{P}$, and the diameter of the describing circle will be $\frac{N}{2P} = \frac{6}{P}$.

Again, if a fifteen-toothed gear be used as the base of the system, the diameter of the describing circle will be $\frac{7.5}{P}$.

Figs. 11 and 12 illustrate a fifteen and a nine-toothed gear engaging a rack. The diameter of the rolling circle by which the teeth were described is $\frac{7.5}{P}$, which will equal 3.75 inches for a 2 pitch gear.

The fifteen-toothed gear will have radial flanks, but the nine-toothed gear will have the flanks much undercut by reason of the diameter of the rolling circle exceeding the radius of the pitch circle.

34. Practical Case of Cycloidal Gearing. PLATE 6. Let F and G be the centers of pinion and gear having twelve and eighteen teeth respectively, and a diameter pitch of 4. The

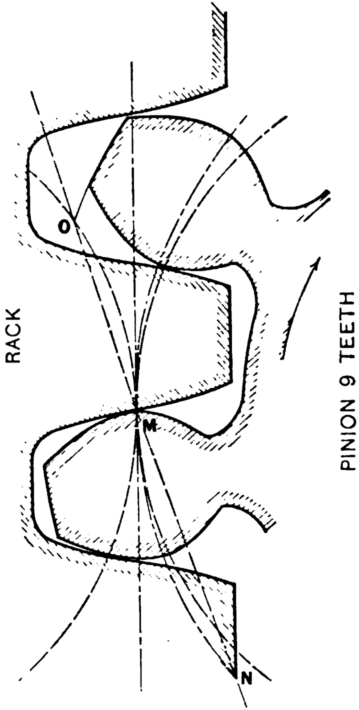


Fig. 12.

pitch diameters will equal $\frac{n}{P} = \frac{12}{4} = 3$ inches, and $\frac{N}{P} = \frac{18}{4} = 4\frac{1}{2}$ inches (ART. 18, page 12). If the tooth be of standard dimensions, the addendum and dedendum lines may be determined and drawn by ART. 31, page 17. The diameter of the rolling circle is assumed to be 1¼ inches for the addendum and dedendum of both gears. Since the teeth should usually be shown in contact at the pitch point, suppose the generating point of the describing curve to be at this point, and describe the curves by rolling the circles from this position, first on the inside of one pitch circle, and then on the outside of the other pitch circle, thus obtaining the flank of one tooth, and the engaging face of a tooth of the other gear.

An enlarged representation of these curves is shown in PLATE 6. They may be drawn by the methods of ARTS. 6 and 7, page 5, or by ART. 9, page 5, but care should be used to draw them in their proper relation to each other, as shown in the figure, so that it may not be necessary to reverse the curves in order to incorporate them into tooth forms. The order for the drawing of the curves may be A B, A T, A D, A S.

Instead of reproducing the tooth curves by means of scrolls, it is sufficiently accurate, and much more rapid, to approximate them by circular arcs. PLATE 2, Fig. 3, illustrates a simple method which closely approximates the curves of this system, and suffices for the ordinary drawing of a gear, but in no case should be used for describing the curves for a templet. This method consists, *first*, in the construction of a normal for a point of the curve at a radial distance from the pitch line equal to two-thirds of the addendum or dedendum of the tooth; *second*, in the finding of a center on this normal, such that an arc may be described through the pitch point, and the point of the tooth already determined. A P is the height of the addendum, and B a point radially distant from the pitch line, equal to $\frac{2}{3}$ A P, through which the arc

B E is drawn. When the point E of the describing curve shall have become a point of contact, as at E′, the arc E′ P being equal to E P, the point P will have moved to T, the chord T E′ being equal to the chord E P. T will be a point in the addendum, and T E′ the normal for this point. From a point, M, on this normal, and found by trial, describe the arc P T, limited by the addendum line. Similarly the curve of the dedendum may be determined.

Having determined such centers as may be required for describing the tooth curves, draw circles through these centers, as indicated in PLATE 6, to facilitate the drawing of other teeth. The radius for the dedendum is often inconveniently great, and in such cases it is desirable to use scrolls, employing the method of ART. 16, page 11. Next divide the pitch circle into as many equal parts as there are teeth, beginning at the pitch point. From each of these divisions lay off the thickness of the teeth. If there be no backlash, this thickness will equal one-half the circular pitch; but if an amount be determined for backlash, the thickness will equal $\frac{P' - \text{backlash}}{2}$.

The circle of centers having been drawn, the tooth curves should be described. These will be limited by the addendum and dedendum circles already drawn. Finally draw the fillets.

The maximum angle of pressure between the pinion and gear will be 24°, the arc of approach .52, the arc of recess .48, and their sum, which is the arc of contact, 1 inch, or 1.27 times the circular pitch.

The rack teeth would be similarly described. The pitch line being a right line, the circular pitch may be laid off directly by scale, or spaced from the pinion. The approximate method may be used for the tooth curves, and lines drawn parallel to the pitch line, for the centers of the arcs which approximate the addenda and dedenda of the teeth.

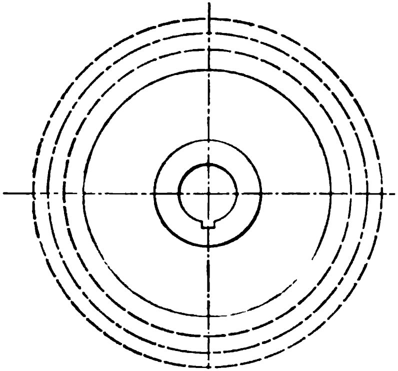


Fig. 13.

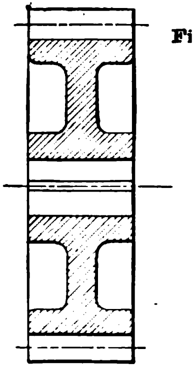


Fig. 14.

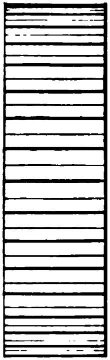


Fig. 15.

35. Face of Gear. In the previous consideration of gear teeth no attention has been paid to the width of the gear, or, as it is commonly termed, the FACE of the gear. This dimension is one of the factors to be considered in determining the strength of the tooth, which is a subject apart from the kinematics of gearing. It should be observed, however, that the tooth having appreciable width, must be generated by an element of a rolling cylinder in place of the point of a rolling circle.

36. Comparison of Gears, illustrated in Plates 4, 5, and 6. In the three cases previously considered, the diameter of the pitch circles are equal, and only one diameter of rolling circle has been used.

In PLATES 4 and 5 the arc of contact is equal to the circular pitch; but the pitch of the latter is twice as great as the former, hence there are but half as many teeth. In PLATE 6 the arc of contact is made dependent on the height of the tooth, which is a standard so chosen as to permit of an arc of contact sufficiently long for a practical case. But in PLATES 4 and 5 the height of the tooth is dependent on the arc of contact, which latter is made the least possible.

The number of teeth in the pinions of PLATES 4 and 6 is the same; but in the former the action is only on one side of the pitch point, there being no addenda to the teeth, hence the limited arc of contact.

In PLATES 4 and 5 there is contact between only one pair of conjugate teeth, save at the instant of beginning and ending contact; while in the case of PLATE 6, two pairs of conjugate teeth may be in contact during a part of the arc of contact.

37. Conventional Representation of Spur Gears. In making drawings of gears, it is usually best to represent them in section, as in Fig. 14. This enables one to give complete information concerning all details of the gear, save the character of the teeth. If the latter be special, an accurate drawing of at least two teeth and a space will be required. Should it be necessary to represent the gears on the plane of their pitch circles, as in PLATE 6, they may be shown as in Fig. 13, thus avoiding the representation of the teeth. Again, if it be necessary to show a full face view of the gears, the method illustrated in Fig. 15 may be employed to advantage. This is simply a system of shading; and no attempt is made to represent the proper number of teeth, or to obtain their projection from another view.

CHAPTER IV.

INVOLUTE SYSTEM.

38. **Theory of Involute Action.** If the describing curves be other than circles we shall obtain odontoids differing in character from those already studied; but so long as both pinion and gear are described by the same rolling curve, the velocity ratio will remain constant. The class of odontoids illustrated by PLATE 7, Fig. 1, is known as the involute, or single-curve tooth. This curve cannot be described by rolling circles, but may be generated by a special curve rolling in contact with both pitch surfaces.* But as the curve may be described by a much more simple process, the above statement is of interest only as showing the conformity of the curve to the general law. (ART. 14, page 10.)

F and G, PLATE 7, Fig. 2, are the centers of two disks designed to revolve about their respective axes with a constant velocity ratio, which is maintained in the following manner: — Suppose the disks to be connected by a perfectly flexible and inextensible band, D C B A, which being wound on the surface of one, will be unwound from the other, after the manner of a belt, producing an equal circumferential velocity in the disks. Conceive a marking point as fixed to the band at A, so that during the motion from A to D, curves may be described on the extensions of disks 1 and 2, in a manner similar to that described for the generating of the cycloidal curves. When the point A, on the band, shall have moved to B, the curve X_1 B will have been described on the extension of disk 2, and $B\,A_1$, on that of disk 1. When the motion of the marking point shall have continued to C, $X_2\,Y_1\,C$ will have been described on the extension of disk 2, and $A_2\,B_1\,C$, on that of disk 1. Finally, when the marking point shall have reached D, the curve $X_3\,Y_2\,Z_1\,D$ will have been described on the extension of disk 2, and $A_3\,B_2\,C_1\,D$ on the extension of disk 1.

---

* For description of this method, see MacCord's Kinematics, page 165, ART. 279.

If these curves be made the outlines of gear teeth, and the former act against the latter so as to produce motion opposed to that indicated by the arrows, a uniform velocity ratio will be maintained between the disks. On investigation, these curves will be found to be involutes, $A_3\,D$ being an involute of the periphery of disk 1, and $X_3\,D$, an involute of disk 2. The curves may, therefore, be described by the method for drawing an involute (ART. 12, page 7), the path of contact, $A\,D$, being spaced off on the base circle from A to A_3, and the involute drawn from A_3; or the line $A\,D$ may be conceived as wrapped about the base circle beginning the curve at D.

39. Character of the Curve. PLATE 7, Fig. 1, represents the involute curve of Fig. 2 incorporated into gear teeth. It becomes necessary to continue the line of the tooth within the periphery of the disk, which will now be designated as the base circle, so as to admit the addenda of engaging teeth. This portion of the tooth is made a radial line.

The pitch point being at B, (the intersection of the line of centers and the line of action), the pitch circles will be drawn through this point.

Base Circle Defined.

The circles from which the involute curves are described, are called *base circles*. Their diameters bear the same ratio to each other as do the diameters of the pitch circles.

The Path of Contact a Right Line.

The line of action, or path of contact, is a right line tangent to the base circles. It is the line followed by the marking point of the model. PLATE 7, Fig. 2.

Constant Angle of Pressure.

Since the path of contact is a right line, and as the common normals at the point of contact must always pass through the pitch point (ART. 14, page 10), it follows that the line of pressure, or angle of the normals, is constant.

Limit of Action

The action between the teeth of the gears in Fig. 1, begins at A, and ends at D, taking place only between the points of tangency of the line of action and base circle. *No involute action can take place within the base circles.*

An Increase in the Center Distance does not Affect the Velocity Ratio.

If the distance between the centers of the gear be increased or decreased, the angle of pressure, and length of the path of contact will be increased or decreased, but the involute curve, which is dependent on the diameter of the base circle only, will remain unchanged. Hence, any change in the distance between the centers of two involute gears will not change the velocity ratio, provided the arc of action is equal to the circular pitch. The case illustrated by Fig. 1 is a limiting one; and therefore an increase in the center distance would mean an increase in the height of the tooth, in order that the arc of action should equal the increased pitch, an increase in the center distance necessitating an increase in the diameters of the pitch circles, and therefore in the circular pitch. But while the action between the teeth continued, the velocity ratio would remain constant.

The Involute Rack tooth, a Right Line.

Since the angle of pressure is constant, and the paths of the elements of a rack tooth are right lines, it follows that the tooth outline of an involute rack must be a right line, perpendicular to the angle of pressure. PLATE 8 illustrates a rack for an involute gear, having an angle of pressure of about 30°. (The section lined portions are not involute.)

40. Involute Limiting Case. PLATE 8. Let the diameters of the pitch circles, the angle of pressure, and the number of teeth, be given. Having drawn the pitch circles about their respective centers, F and G, obtain the base circles as follows: —

Through the pitch point, B, draw A D, making an angle with the tangent at the pitch point equal to the angle of pressure. This will be the line of action; and perpendiculars, F A and D G, drawn to it from centers F and G, will determine the radii of the base circles, and the limit of the action, or path of contact, at A and D. This is a limiting case, in that the path of contact is a maximum, and the arc of contact equal to the circular pitch. Next determine the point, C, by spacing the arc, D K C, equal to D A; A and C will be two points in the involute curve of the base circle, D K C, from which other points may be obtained. Similarly describe D P, the involute of the other base circle, just beginning contact at D. The height of the teeth will be limited by the addendum circles drawn through D and A, from centers, F and G. The dedendum circles are made to admit the teeth without clearance. The pinion teeth are pointed, and the gear teeth fill the space, having no backlash. The circular pitch may be found by dividing the circumference of the pitch circle into as many parts as there are teeth, or the teeth may be spaced on the base circle.*

The rack is made to engage the pinion in the following manner: —

O being the pitch point of rack and pinion, the right line, O R, drawn through this point, and tangent to the base circle, will be the path of contact for motion in the direction indicated by the arrow. The contact will begin at R and end at S, the latter point being that of the intersection of the path of contact and addendum circle. The rack tooth will be perpendicular to the line of action, R S; and the thickness of tooth will equal that of the gear tooth, there being no backlash in either case. The addendum of the rack tooth will be limited by the parallel to the pitch line drawn through the first point of contact, R; and the dedendum made sufficiently great to admit the pinion tooth without clearance.

---

* For further details concerning the construction of this pinion and gear, see Problem 4, Page 74.

41. Epicycloidal Extension of Involute Teeth. The extent of the involute action between the gear and the pinion of PLATE 8 is limited to the path D A; for while an increase in the height of the gear tooth is possible, the limit of the engaging involute tooth is at A, since no part of an involute curve can lie within its own base circle. It is, however, entirely feasible to continue the contact by a cycloidal action, in the following manner: —

The angle F A B being a right angle, the circle described on F B as a diameter must pass through A. This point may therefore be considered as a point in an epicycloid, described by the rolling circle F A B, and having A B for its normal, which is also the normal for the involute. But this diameter of rolling circle being one-half the pitch circle within which it rolls, the hypocycloid will be a radial line, and the dedenda of the teeth will be radial within the base circle. By rolling the same circle on the outside of the gear pitch circle, the addenda of the gear teeth may be extended, and the path of contact continued to N, which is a limit in this case, by reason of the gear tooth having become pointed.

Similarly, the addendum of the rack tooth may be extended by the same describing circle. In the figure it is made sufficiently long to just clear the dedendum circle required for the pointed gear tooth. The action will now begin at Q, follow the rolling circle to R, and then, becoming involute, continue to S.

42. Involute Practical Case, PLATES 9 and 10. Having given the number of teeth of engaging gears, and the diameters of their pitch circles, it is required to determine the curves for the involute teeth of a pinion, gear, and rack.

The diameters of the describing circles would be of first consideration in cycloidal gearing; while in the involute system, the angle of pressure or line of action must first be established; and tangent to this the base circles may be drawn. By reference to PLATE 7, Fig. 1, it will be seen that with a constant center distance, a decrease in the angle of pressure will necessitate an increase in the diameter of the base circles, and a corresponding decrease in the path of contact. That is to say, an increase in the possible length of the path of contact means an increase in the angle of pressure. In PLATE 7, Fig. 1, this angle is too great for actual practice, being about 30°; yet it cannot be lessened in this case, as the number of teeth is limited. Practice has limited this angle to 14½° or 15°, which is, unfortunately, too small; but as one of these angles is generally adopted in the manufacture of gears, the latter will be used in the following problem: —

A pinion of 12 teeth is required to engage a gear of 30 teeth, and a rack, the diameter pitch being 1. The former is illustrated by PLATE 9, and the latter by PLATE 10.

$$\text{Pinion diameter} = d' = \frac{n}{P} = \frac{12}{1} = 12 \text{ inches.}$$

$$\text{Gear diameter} = D' = \frac{N}{P} = \frac{30}{1} = 30 \text{ inches.} \quad \text{(ART. 18, page 11.)}$$

Since the teeth are to be of standard dimensions (ART. 31, page 17), the addenda will be 1 inch, the dedenda 1⅛ inches; and there being no backlash, the thickness of the teeth will be half the circular pitch, or $\frac{P'}{2}$. The circular pitch, P', $= \frac{\pi}{P} = 3.1416$. Draw the pitch circles. The line of action will pass through the pitch point, making the required angle with the common tangent at this point. Next draw the base circles tangent to this line, and determine the points of tangency, D and A. Construct the involutes of these base circles in the manner

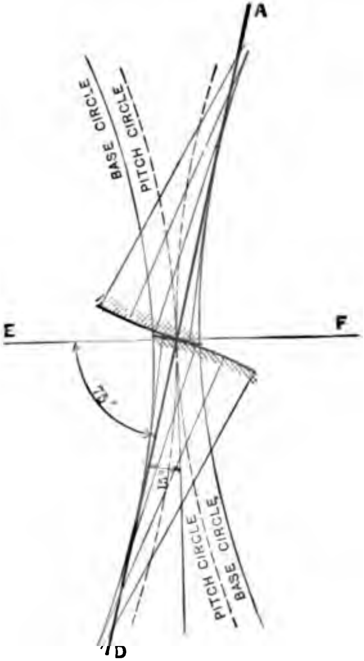


Fig. 16.

indicated by Fig. 16, and according to the method for describing an involute, Art. 12, page 7. It will now be seen that the gear tooth will be limited by the arc drawn through D, the point of tangency of base circle and line of action. If, however, the involute curve be continued to the addendum circle, as shown by the dotted line, C E, it will interfere with the radial portion of the pinion flank, which lies within the base circle. The pinion tooth will have no such limitation, since the addendum circle intersects the line of action, D A, at L, a considerable distance from the limit of involute action, at the point A.

Similarly, the rack tooth will be found to interfere with the pinion flank, if extended beyond the point C, which comes into contact at the point D, the limit of involute action. But the pinion face may be extended indefinitely, so far as involute action is concerned. The remedy for this interference is treated of in the following article.

43. Interference. Since practical considerations demand the maintenance of a standard proportion of tooth, two schemes are adopted for avoiding or correcting this interference, observed in Plates 9 and 10.

The first is to hollow that part of the pinion flank lying within the base circle so as to clear the interfering part of the gear, or rack tooth. In this case there will be no action beyond the point of tangency D. The second method consists in making the interfering portion of the addendum an epicycloid described by a circle of a diameter equal to the radius of the pinion pitch circle. Such a describing circle would generate a radial flank for that part of the curve lying within the base circle. By this means, the action will be continued and the velocity ratio maintained, although the action will cease to be involute. ART. 41, page 30.

44. Influence of the Angle of Pressure. The interference may be entirely obviated by sufficiently increasing the angle of pressure; but in the case cited (PLATES 9 and 10) it would necessitate an angle of 24.1°, which is too great for general use. Had the number of teeth in the pinion been greater, the interference would have been less, and with 30 teeth in the pinion, there would have been no interference. See ART. 45.

The angles of 14½° and 15°, commonly adopted, are unfortunately small. There is, however, a tendency to increase this angle, and gears for special machines have been made with a 20° angle of pressure. This latter angle will permit gears having 18 teeth to engage without interference, and the thrust due to this increase in the angle of pressure is an insignificant amount. A system based on this angle of pressure would unquestionably be an improvement over the present one.

45. Method for determining the Least Angle of Pressure for a Given Number of Teeth having no Interference. Fig. 17.

Let A be the center of a gear having A B = R for the radius of pitch circle, and D B T the

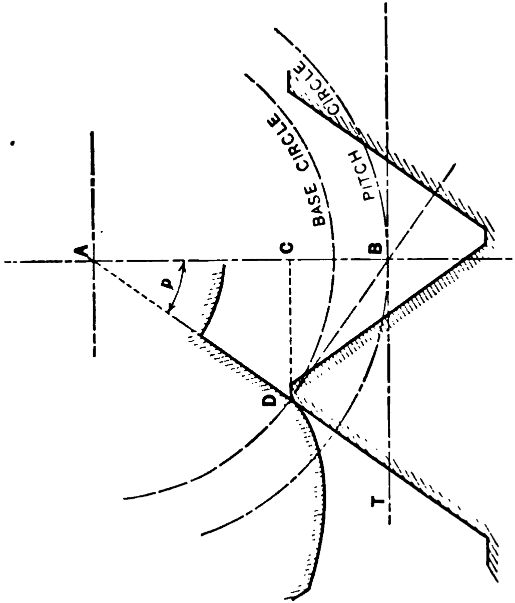


Fig. 17.

angle of pressure to be determined, the least number of teeth being N. Suppose the gear to engage a rack having standard teeth, then will $BC = \frac{1}{P} = \frac{D}{N} = \frac{2R}{N}$. D will be the last point of contact, and $AD = r$, the radius of the base circle.

$$AC = AB - BC = R - \frac{1}{P} = R - \frac{2R}{N} = \frac{R(N-2)}{N}.$$

$$AC : AD :: AD : AB, \text{ hence, } \overline{AD}^2 = r^2 = AB \times AC = \frac{R^2(N-2)}{N} \text{ and } r = R\sqrt{\frac{N-2}{N}}.$$

The angle of pressure, DBT, is equal to angle $DAB = p$, and

$$\text{the cos. } p = \frac{r}{R} = \frac{R\sqrt{\frac{N-2}{N}}}{R} = \sqrt{\frac{N-2}{N}}.$$ Hence the

$$\text{cos. of the angle of pressure} = \sqrt{\frac{N-2}{N}} \quad (5).$$

By substituting in the above formula, it will be seen that for a 12-toothed gear to engage a rack without interference, the angle of action must be 24.1°, and for 15 teeth the angle would be 21.4°. Again, if the angle be 15°, the least number of teeth that will engage without interference will be 30, while with a 20° angle of pressure the least number would be 18.

46. Defects of a System of Involute Gearing. As in the case of the cycloidal system, it is desirable to make all involute gears having the same pitch to engage correctly. In cycloidal gears this was attained by the use of one diameter of rolling circle for all gears of the same pitch (ART. 33, page 20). In the involute system we assume an angle of obliquity, or pressure, which is constant for all gears; but unless this angle be great, gears having so few as 12 teeth cannot be run together without interference. To obviate this difficulty we must adopt one of the two methods already described (ART. 43, page 32); namely, the undercutting of the interfering flanks, or the rounding of the interfering addenda. First consider the latter, which is illustrated by PLATES 9 and 10. We have seen how that portion of the gear tooth addendum lying beyond the point C must be made epicycloidal in order to engage the radial part of the pinion flank which lies within the base circle; also that the pinion addenda might be wholly involute since there would be no interference with the gear tooth flank, the action between the latter taking place without the base circle. But if a 12-toothed gear be taken as the base of the system, it will be necessary to round, or epicycloidally extend that portion of the pinion addendum lying beyond the point K, since this would be the last point of involute action between two 12-toothed gears. Therefore when the 12-toothed gear engages one having a greater number of teeth, that part of the addendum lying beyond this point will no longer engage the second gear, and the arc of contact will be greatly reduced. Again, suppose a pair of 30-toothed gears to engage (each being designed to engage a 12-toothed pinion), the only part of the tooth suitable for transmitting a uniform motion is that lying between the base

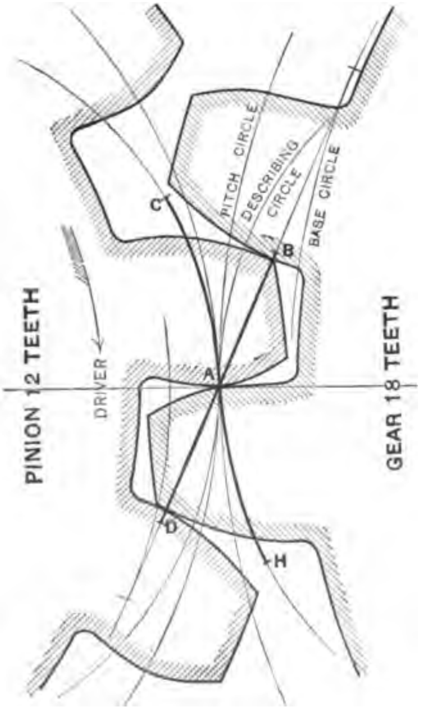


Fig. 18.

circle and point C, Plate 9, and the arc of contact would be but 1.05 of the circular pitch. Now, one of the claims made for the involute tooth is that the distance between the centers of the gears may be changed without changing the velocity ratio; but in this latter case it cannot be done without making the arc of contact less than the circular pitch.

If the system of undercutting the flanks be adopted, the addendum will be wholly involute; and in the case of Plate 9 all of the pinion addendum would have been available for action, but the pinion flank, within the base circle would have been cut away so that there would have been no action of the gear addendum between C and E. If, however, the engagement had been between two 30-toothed gears, all of the tooth would have been available for action, and the arc of contact would have been equal to 1.91 of the circular pitch.

Thus it will be seen that involute gears should be designed to engage the gears with which they are intended to run, if the best results would be attained. This would, of course, prevent the use of the ready-made gear or cutter, but would insure a longer arc of action between conjugate teeth.

47. Unsymmetrical Teeth. Fig. 18. A very desirable, although little used, form of tooth is that known as the unsymmetrical tooth, which usually combines the cycloidal and involute systems. Fig. 18 illustrates a pinion and gear having the same number of teeth as those illustrated by PLATE 4, and the arc of contact is unchanged; but the angle of pressure is much reduced, and the strength of the tooth increased. As the involute face of the tooth is designed to act only when it may be necessary to reverse the gears, and when less force would usually be transmitted, the angle of pressure may be made greater than ordinary. In this case the angle is 24.1°, which is sufficient to avoid interference in a standard 12-toothed gear (ART. 45, page 33). But this angle is no greater than the maximum angle of pressure in PLATE 4. This reinforcement of the back of the tooth makes it possible to use a much greater diameter of rolling circle; and in the case illustrated, the diameter is one-third greater than the radius of the pitch circle. This increase in the diameter of the rolling circle would have lengthened the arc of contact, had not the height of the tooth been reduced to maintain the same arc as that of PLATE 4.

The cycloidal action begins at C and ends at H, making a maximum angle of pressure of 17°. The same rolling circle has been used for the face and flank of each gear; but the one rolling within the pitch circle of the gear might have been much increased without materially weakening the gear tooth.

The involute action would begin at D and end at B, making an arc of contact a little greater than the pitch.

CHAPTER V.

ANNULAR GEARING.

48. Cycloidal System of Annular Gearing. If the center of the pinion lies within the pitch circle of the gear, the latter is called an *internal*, or *annular gear*. The solution of problems relating to this form of gearing differs in no wise from that of the ordinary external spur gear, save in the consideration of certain limitations which will be treated of.

49. Limiting Case. PLATE 11 illustrates a pinion engaging an internal and an external spur gear. The pinion has 6 teeth, and the gears have 13 teeth. The arc of contact is made equal to the circular pitch, and equally divided between recess and approach. The pinion has radial flanks, which therefore determines the diameter of the describing circle for the addenda of the gears. The second describing circle, 2, is governed by conditions which will appear later. It will be observed that the addenda of the annular gear teeth lie within, and the dedenda without, the pitch circle. The height of the teeth is governed by the arcs of approach and recess; and the construction of the teeth does not differ from the limiting case considered in ART. 26, page 14, and PLATE 5. The action between the pinion and annular gear begins at B, and ends at C, the pinion driving.

50. Secondary Action in Annular Gearing. We have already seen, ART. 10, page 6, that every epicycloid may be generated by either of two rolling circles, which differ in diameter by an amount equal to the diameter of the pitch circle. Also, that every hypocycloid may be generated by either of two rolling circles, the sum of the diameters of which shall equal that of the pitch circle within which they roll. Thus the addendum, C E, of the pinion, PLATE 11, may be described by the circle 2, or the intermediate circle 3. But in this case the circles 1 and 2 are so chosen that the intermediate circle 3 is the second describing circle for the hypocycloid F G, as well as for the epicycloid C E; consequently C E and F G will produce a uniform velocity ratio, the contact taking place from A to D. The addendum C E has contact also with the dedendum C F along the path A C; hence, during a part of the arc of recess there must be two points of each tooth in contact at the same time.

The plate illustrates the contact along the path A C as just completed; but a second point of contact will be seen on circle 3, between F and E, and action along this path will be continued to D. The case is therefore no longer a limiting one, inasmuch as the arc of contact is greater than the circular pitch. The additional contact takes place during the arc of recess, which is also advantageous.

In order to obtain this secondary action, *the sum of the radii of the inner and outer rolling circles must equal the distance between the centers of pinion and gear.*[*]

For, letting r_1, r_2, and r_3 be the radii of the inner, outer, and intermediate rolling circles, and R_p, R_g, the radii of pinion and gear, $r_3 + r_1 = R_g$, (6), and $r_3 - r_2 = R_p$, (7), ART. 10, page 6. Subtracting the second equation from the first, $r_1 + r_2 = R_g - R_p = C =$ center distance (8).

PLATE 12, Fig. 1, illustrates the same pinion and gear, the teeth having been described by the intermediate circle only. In this case the action takes place wholly during recess, the arc of recess being the same as before, about 1¼ times the circular pitch. Had the outer describing circle been used to describe the dedenda of the gear teeth, as in the preceding cases, a secondary action would have taken place during the recess.

---

[*] The student is referred to Prof. MacCord's "Kinematics," pages 104 to 109 inclusive, for a very complete demonstration of this law, together with other limitations of annular gears.

Special notice should be taken of the reduced angle of pressure in the secondary action of annular gearing, and of the possibility of obtaining a great arc of recess with little or no approaching action. These advantages are very apparent in PLATE 11, in which the pinion engages an external and an internal gear having an equal number of teeth.

51. Limitations of the Intermediate Describing Circle. PLATE 12, Fig. 2. Suppose the inner describing circle, 1, PLATE 11, to be increased until it equals the diameter of the pinion pitch circle, 9¾, the radius of the intermediate describing circle will then equal the center distance, $5\frac{1}{16}$, and the outer describing circle, 2, would be but $\frac{3}{16}$ radius. For by substituting Rp for r_1 in equations 6 and 8, ART. 50, we shall obtain $r_3 = Rg - Rp = C$, and $r_2 = C - Rp$. PLATE 12, Fig. 2, illustrates this case, the outer describing circle not being employed.

Since the pinion pitch circle has now become a describing curve, there will be an approaching action; but only one point of the pinion tooth will act, as the diameter of the describing circle and pitch circle being equal reduces the pinion flank to a point. But if any further increase be made in the diameter of the inner circle, which is equivalent to a decrease in the intermediate describing curve, an interference will take place during approaching action; since the curves of gear and pinion teeth, generated by a circle greater than the pinion diameter, will cross one another, which would make action impossible. Hence, *the radius of the intermediate describing circle cannot be less than the line of centers.*

52. Limitations of Exterior and Interior Describing Circles. PLATE 12, Fig. 3. From

ART. 50, page 39, it was seen that the sum of the radii of the exterior and interior describing circles must equal the center distance if a secondary action be obtained. If either circle be decreased without decreasing the other, the secondary action ceases; but if either circle be increased without an equal decrease in the other, thus making the sum of their radii greater than the center distance, the addenda will interfere. Thus, in PLATE 11, a decrease in describing circle 2 would produce a more rounding face, and C E would fail to engage F G; but had this describing circle been increased in diameter without a corresponding decrease in 1, C E would have interfered with F G. Hence, *the limit of the sum of the radii of the exterior and interior describing circles is the center distance.*

PLATE 12, Fig. 3, illustrates a special case of the above condition, the interior describing circle being reduced to zero, and the radius of the exterior circle made equal to the center distance, thus making the intermediate describing circle equal to the pitch circle of the gear. There will be double contact during a portion of the arc of recess, the contact beginning at A, and following the outer describing circle to C, and the intermediate (or in this case the pitch circle of the gear) to D. This design is objectionable in that the secondary action takes place with only one point of the gear tooth.

53. The Limiting Values of the Exterior, Interior, and Intermediate Describing Circles for Secondary Action. Since $r_2 + r_1 = C$, either radius will equal C, when the other becomes zero; but if there be a secondary action, the minimum value of r_2 may not be zero, for r_1 will be a maximum when r_3 is a minimum, as $r_1 + r_3 = Rg$, r_3 is a minimum when equal to C (ART. 52), and substituting this value in the last equation, $r_1 = Rg - C$. Again substituting this value in the equation, $r_2 + r_1 = C$, $r_2 = C - (Rg - C) = 2C - Rg$.

Summary of the above limiting values and conditions governing secondary action:—

$$r_1 \text{ maximum} = Rg - C; \qquad r_1 \text{ minimum} = 0; \qquad r_3 + r_1 = Rg. \qquad (6)$$

$$r_2 \text{ maximum} = C; \qquad r_2 \text{ minimum} = 2C - Rg; \qquad r_3 - r_2 = Rp. \qquad (7)$$

$$r_3 \text{ maximum} = Rg; \qquad r_3 \text{ minimum} = C; \qquad Rg - Rp = C. \qquad (8)$$

54. Practical Case. If annular gears be made interchangeable with spur gears, it will be necessary to have the number of teeth in the engaging gears differ by a certain number which will depend on the base of the system. This is due to the limitation in the sum of the radii of the describing circles, ART. 52, page 40. Thus, let 12 be the base of the system, and it is required to find the least number of teeth in the annular gear that will engage the pinion. If the pitch be 2, the diameter of the pinion will be 6, and that of the describing circles 3. But since the center distance cannot be greater than the sum of the radii of the describing circles (in this case 3), the diameter of the annular gear must be 12, and the least number of teeth in the annular gear will be 24.

Using the notation of PLATE 11, and ART. 17, page 11, N being the least number of teeth in the gear, and n the least number in the pinion, or the base of the system:—

$$C = 2r_1 = \frac{n}{2P}, \text{ also } C = Rg - Rp = \frac{N}{2P} - \frac{n}{2P}, \text{ hence } \frac{n}{2P} = \frac{N}{2P} - \frac{n}{2P}, \text{ or } 2n = N.$$

The least number of teeth in the annular gear will be twice that of the base of the system.

55. Summary of Limitations and Practical Considerations. (*a*) The diameter of the intermediate describing circle is equal to the diameter of the pinion, plus the diameter of exterior describing circle, or diameter of gear minus interior describing circle. (ART. 10, page 6.)

(*b*) There will be secondary action only when the sum of the radii of the exterior and interior describing circles is equal to the line of centers. (ART. 50, page 38.)

(*c*) The radius of the intermediate describing circle cannot be less than the center distance. (ART. 51, page 40.)

(*d*) The sum of the radii of exterior and interior describing circles cannot be greater than the center distance. (ART. 52, page 40.)

(*e*) The number of teeth in any pair of gears of an interchangeable system must differ by an amount equal to the base of the system. (ART. 54, page 42.)

(*f*) If the pinion drives, the exterior describing circle should be the greater in order that the arc of contact may be chiefly one of recess.

(*g*) If the gear drives, the interior describing circle should be the greater, and the pinion teeth may have flanks only, but in this case the teeth should be extended slightly beyond the pitch circle in order to protect the last point of contact, which will be on the pitch circle.

56. Involute System of Annular Gearing. Fig. 19. The method of drawing the tooth outlines for the involute annular gear does not differ from that of the spur gear. Pitch lines having been determined, the base circles are drawn tangent to the line of action, and the involutes of those base circles will be the required curves. Care must be used in obtaining the length of the teeth, in order to avoid a second engagement after the full action shall have taken place. To determine if this interference takes place, it is necessary to construct the epitrochoid of the point of the pinion tooth, or determine the path of least clearance, as in ART. 28, page 15.

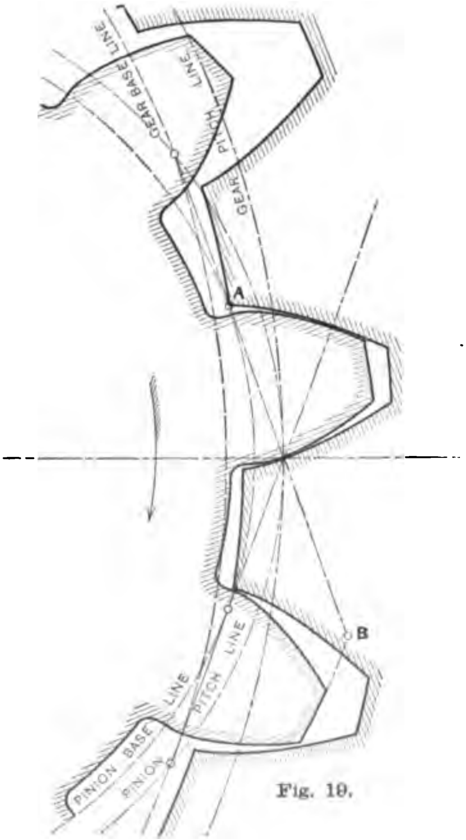


Fig. 19.

Fig. 19 illustrates an annular gear of 20 teeth engaging a pinion of 10 teeth, the angle of pressure being 20°. The pinion driving in the direction indicated will establish the first point of contact at A, and the last point, B, will be limited by the height of the tooth, in this case $\frac{1}{P}$. The limit of the gear tooth will be determined by the arc drawn from the center of gear through the point A. Any extension of the involute beyond this point will interfere with the pinion flank. The stronger form of the annular gear tooth permits of a greater clearance, which it is advantageous to adopt.

If the pinion and gear differ but little in diameter, it is desirable to use the cycloidal system, in which case the interference may be more easily avoided. It should also be noted that the advantages to be derived from an increase in the arc of contact and a decrease in the angle of pressure are only to be obtained by the use of the latter system.

CHAPTER VI.

BEVEL GEARING.

57. Theory of Bevel Gearing. In all cases previously considered, the elements of the teeth were parallel, the surfaces having been generated by a right line which was either an element of a rolling cylinder, as in the cycloidal system, or by an element of a flexible band parallel to the axis of a cylinder from which it was unwrapped, as in the involute system. All sections of the teeth made by planes perpendicular to the axis were alike, and therefore it was only necessary to consider one. Under these conditions the pitch cyclinder became a pitch circle, and the describing cylinder a describing circle. If we now consider the axes of the gears as intersecting, the friction cylinders will become friction cones, the describing cylinder will be a describing cone, and the elements of the teeth will converge to the point of intersection of the axes, making all sections of the teeth to differ from one another.

Fig. 20, page 46, illustrates this case. A C B and B C D are two friction cones, or pitch cones, having axes G C and H C. The outlines of the teeth are drawn on the spherical base of the cone, that portion of the curve lying outside the pitch cone being a spherical epicycloid, and that within, a spherical hypocycloid. The dedendum, or surface of the tooth lying within the pitch cone A C B, was described by the element E F C of the describing cone, which is shown as generating the addendum of the pinion tooth. Only that portion of the surface described by E F would be used for the pinion tooth, the length of the gear tooth having been limited as shown. The describing cone employed for generating the addendum of gear, and dedendum of pinion, is not shown; but the diameter of its base would be governed by laws similar to those already considered for limiting the diameters of rolling circles, Art. 32, page 18.

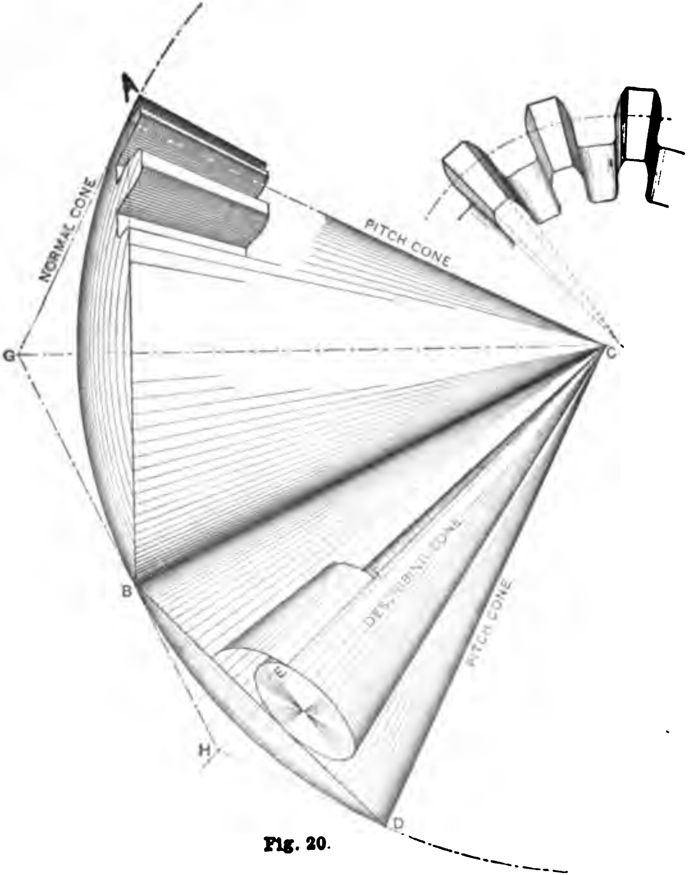


Fig. 20.

58. Character of Curves employed in Bevel Gearing. The CYCLOIDAL BEVEL TOOTH has already been considered in the previous article, and the curve does not differ from that employed in spur gearing, save that it is described on the surface of a sphere.

It is important to note that no tooth can be made with a radial flank, since no circular cone can be made to generate a plane surface by rolling within another cone, but the flank may approximate closely to such plane.

The INVOLUTE BEVEL TOOTH is one having a great circle for its line of action. Fig. 21 illustrates a crown gear of this type. A C is a great circle of the sphere A D C E, and is tangent to the circles A E and D C. If the circle A C be rolled on D C, so as to continue tangent to D C and A E, the point B will describe the spherical involute G D F. Conjugate teeth described by this process maintain their velocity ratio constant, even while undergoing a slight change in their shaft angles, thus conforming to the general character of involute curves.

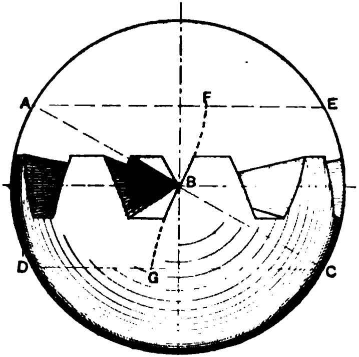


Fig. 21.

The OCTOID BEVEL TOOTH is one having a plane surface for the addendum and dedendum, the plane being such as would cut a great circle from the surface of the sphere. In Fig. 22, G F is the plane which cuts the surface of the tooth shown at B. The line of action, from which the tooth takes its name, is indicated by the curve B C E B H K. This tooth was the invention of Hugo Bilgram, and is of interest in being the only bevel tooth that can be formed in a practical manner by the molding-planing process. The Bilgram machine, designed to plane this tooth, is described in the *Journal of the Franklin Institute* for August, 1886, and in the *American Machinist* for May 9, 1885.

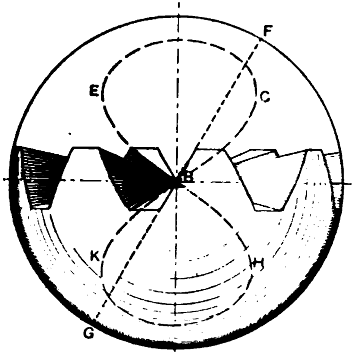


Fig. 22.

59. Tredgold Approximation. Because of the difficulty involved in describing the tooth form on the surface of a sphere, it is customary to draw the outline on the developed surface of a cone which is tangent to the sphere at the pitch circle. This cone is called the normal, or back cone. PLATE 13 illustrates a sphere A B D, from which the pitch cones A C B and B C D have been cut. Tangent to the sphere at the pitch circles, A B and B D, are the normal cones A G B and B H D, the elements of which are perpendicular to the intersecting elements of the pitch cones. The error in the tooth curve due to this approximation is so small as to be inappreciable, save in exaggerated cases; and the method is always employed for the drafting of bevel gears.

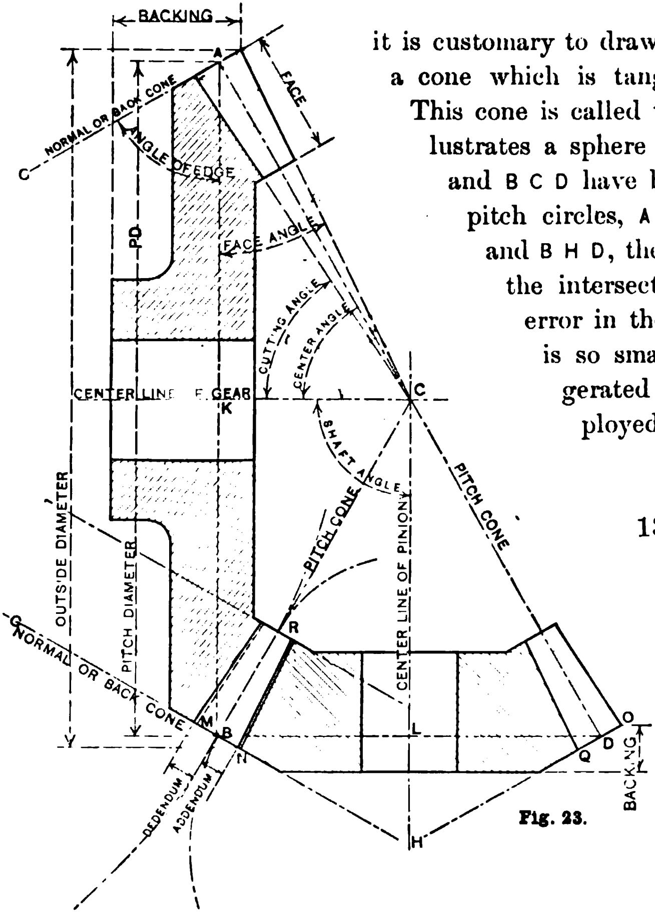


Fig. 23.

60. Drafting the Bevel Gear. PLATE 13, and Fig. 23. The drawing usually required is that illustrated by Fig. 23, which is a section of a gear and pinion, together with the development of a portion of the outer and inner normal cones, only the tooth curves being omitted.

The names of the parts of a bevel gear are also given, and the lettering corresponds to that of PLATE 13, which latter will be used to illustrate the method of drawing.

A B and B D, PLATE 13, are the pitch diameters of a gear and pinion with axes at 90°, and having 15 and 12 teeth respectively, the pitch being 3, when drawn to the scale indicated. The pitch diameters being 5″ and 4″, lay off C K on the center line of gear, equal to one-half the pitch diameter of pinion, and C L on the center line of pinion, equal to one-half the pitch diameter of the gear. Through these points draw the pitch lines perpendicular to the axes of the gears, and in this case perpendicular to each other. Draw the pitch cones A C B and B C D, and perpendicular to these elements draw G A, G B H, and H D, elements of the normal cones. Having figured the addendum and dedendum of the teeth, lay off on the normal cone of pinion B M and B N, D O and D Q, and from these points draw lines converging to the apex of the pitch cones. Similarly lay off addenda and dedenda of gear, limiting the length of the face at R by drawing the elements of the inner normal cones at R S and R T. The face B R should not be greater than one-third B C, by reason of the objectionable reduction in small end of teeth. Complete the gear blank, or outline, by drawing the lines limiting the thickness of the gear, diameter and length of hub, diameter of shaft, etc., details which are matters of design.

The development of the normal cone of the gear, B G A, will be a circular segment described with radius G B, and equal in length to the circumference of the pitch circle of the gear. Since there are 15 teeth in the gear, the developed pitch circle will be divided into 15 parts, as shown, and the circular pitch be thus determined. But it is unnecessary to obtain the complete development as shown in the plate, since the shape of one tooth and space is alone required. Therefore, space off on a portion of the arc of the developed pitch circle, the circular pitch, B V, which is equal to $\frac{\pi}{P}$. Draw the addendum and dedendum circles with radii equal to distance of these circles from the apex of the normal cone, which in the case of the gear will be G E and G F.

Next determine the tooth curve as for spur gears, using the developed pitch circle instead of the real pitch circle. In the case illustrated, the curve is involute. B W is a part of the line of action, making an angle of 75° with G H, the line of centers. The base circles drawn tangent to this line will be the circles from which the involutes are described. Had the cycloidal system been employed, the diameter of the rolling circle would have been made dependent on the diameter of the developed pitch circle, instead of the pitch diameter A B.

In like manner obtain the development of the inner normal cones, having S R and T R for elements, and describe the true curves of the small end of teeth. These pitch circles may be drawn concentric with the developed pitch circles of the outer cones, or with S and T as centers, the latter being the method commonly adopted. Both methods have been employed in the plate. If the development of the inner pitch cone of gear be drawn from the center G, the reduced pitch, and thickness of tooth, may be obtained by drawing the radial lines from the development of the outer cone as shown by the fine dotted lines. The addendum and dedendum circles will be described with radii S Z and S Y, and the tooth curves may be drawn by determining the reduced rolling circle, if the gear be cycloidal, or the reduced base circle if the involute system be employed.

A second method for describing the teeth on the inner normal cone would be to base it directly on the reduced pitch, which may be determined by dividing the number of teeth by the diameter of the base of the pitch cone at this point. In the plate, the value of P for small end of teeth is $\frac{15}{3.24}$ for gear, or $\frac{12}{2.59}$ for pinion $= 4 6 = P$. The addendum, dedendum, circular pitch, etc., may now be obtained from this value of P, as was done in the case of the outer pitch cone. In like manner we may obtain any other section of the tooth, although a third section is seldom required.

61. Figuring the Bevel Gear with Axes at 90°. Figs. 24 and 25. The dimensions required for the figuring of a pair of bevel gears will be: —

First: Those required for general reference, and consisting of pitch diameters, number of teeth (or pitch), face (K), thickness of gears (L and M) (U and V), diameter and length of hubs.

Second: In addition to the above, the pattern maker and machinist will require, for the turning of the blank, the outside diameter, backing, angle of edge, angle of face.

Third: The cutting angle will be required for cutting the teeth.

The figures required for the first set of dimensions are all matters of design, but the second and third dimensions must be

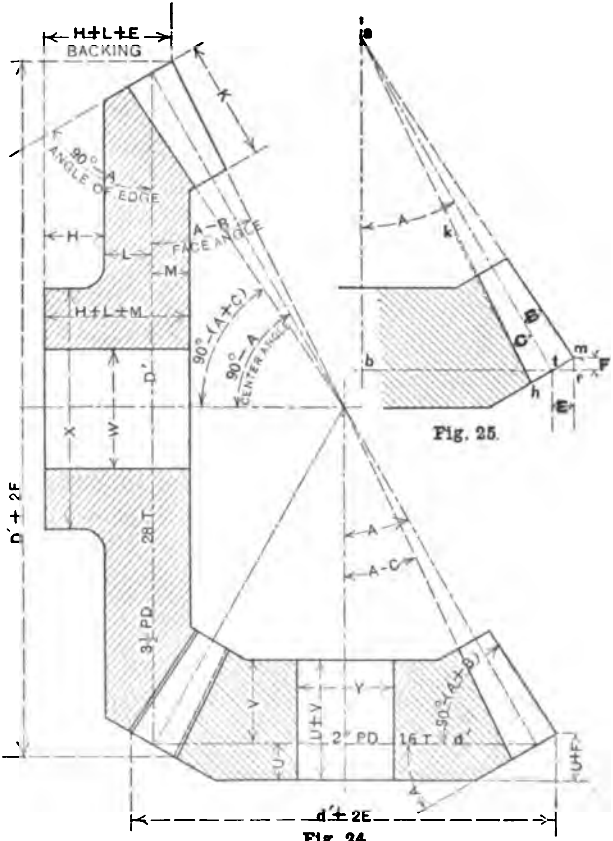


Fig. 25.

Fig. 24.

determined from the data given in the first. To obtain these it is necessary to figure the five dimensions indicated in Fig. 25, three of which, A, B, and C, are angles, and two, E and F, are necessary to determine the outside diameter and backing. Only one of these, A, is used directly. B is called the angle increment, C the angle decrement, E is one-half the diameter increment of the pinion, and F is equal to one-half the diameter increment of the gear.

In the similar right triangles a b t and t r m, Fig 25,

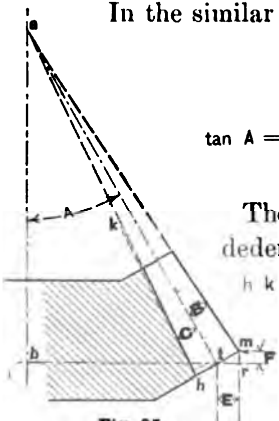


Fig. 25.

$$t\,a\,b = m\,t\,r = A. \qquad a\,b = \frac{D'}{2}. \qquad b\,t = \frac{d'}{2}. \qquad t\,m = \frac{1}{P}.$$

$$\tan A = \frac{d'}{D'} = \frac{n}{N}. \qquad \tan B = \frac{\frac{1}{P}}{\frac{d'}{2\sin A}} = \frac{2\sin A}{P\,d'} = \frac{2\sin A}{n}. \qquad E = \frac{1}{P}\cos A. \qquad F = \frac{1}{P}\sin A.$$

The angle decrement, C, is sometimes made equal to B, in which case the dedendum of the tooth at the small end will be greater, as shown by the line h k; but if the bottom line of the tooth be made to converge to the apex of the pitch cones, the angle t a h, or C, will be determined as follows:

$$\tan C = \frac{h\,t}{t\,a} = \frac{\frac{\frac{9}{8}P}{d'}}{\frac{2}{\sin A}} = \frac{9}{4} \bullet \frac{\sin A}{n} = \frac{2.25\sin A}{n}.$$

Having determined these values, it is only necessary to combine them with those fixed by the design to complete the figuring of the gear as shown in Fig. 24.

The angles should be expressed in degrees and tenths, rather than in degrees and minutes. It is also of importance that the outside diameter and backing be figured in decimals, to thousandths, rather than in fractional equivalents.

62. Bevel Gear Table for Shafts at 90°. In order to facilitate the figuring of bevel gears, tables or charts of the principal values are commonly employed. Such charts also make the figuring possible to those unfamiliar with the solution of a right triangle. Some are designed to solve the problems graphically, while others, like the following, pages 54 and 55, consist of the trigonometrical functions for gears of the proportions commonly employed.

DESCRIPTION OF TABLE.*

COLUMN 1. Ratio of Pinion to Gear.

COLUMN 2. Ratio of Pinion to Gear expressed in decimals, or tang of center angle. $\tan A = \frac{d'}{D'} = \frac{n}{N}$.

COLUMN 3. Center angle of Pinion corresponding to tangent in column 2.

COLUMN 4. Ten times the angle increment for a Pinion of 10 teeth. This increased value is employed to simplify the figuring of gears having other than 10 teeth. Thus, the angle increment for miter gears (1 to 1) having 10 teeth would be 8.2°, and for 14 teeth, $\frac{10}{14}$ of this value or $\frac{8.2}{14}$. There is, of course, a slight error in deriving the angle increment for any number of teeth from these values, in that the tangent and arc do not vary alike, but the error is inappreciable for small arcs.

COLUMN 5. The diameter increment for a Pinion of one pitch, hence equal to 2 cos A.

COLUMN 6. Center angle for Gear, or 90° – A.

COLUMN 7. Ten times the angle increment for Gear of 10 teeth, which, of course equals that of the engaging Pinion.

COLUMN 8. Diameter increment for a Gear of one pitch, hence equal to 2 sin A.

USE OF TABLE. — In columns 1 or 2 find the value corresponding to the ratio of given gears. Against this value, in 3 and 6, the center angles for pinion and gear are given.

The angle increment may be found by dividing the value in 4 by the number of teeth in the pinion, or by dividing the value in 7 by the number of teeth in the gear.

The diameter increment for the pinion is obtained by dividing the value in 5 by P, and that for the gear by dividing the value in 8 by P.

The value of the angle B may be determined with sufficient accuracy by making it $\frac{8}{9}$ of B.

---

[*] The plan of this table is that adopted by Mr. George B. Grant. See "American Machinist," Oct. 31, 1885, and "Odontics," page 90.

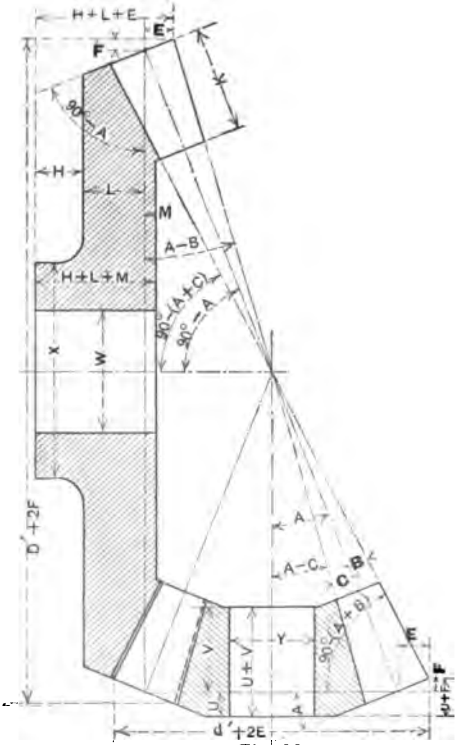


Fig. 26.

BEVEL GEAR TABLE FOR SHAFTS AT 90°.

PROPORTION OF PINION TO GEAR.		PINION.			GEAR.		
		A.	B.	2 E.	90° — A.	B.	2 F.
		Center Angle.	Angle Increment. Divide by n.	Diameter Increment. Divide by P.	Center Angle.	Angle Increment. Divide by N.	Diameter Increment. Divide by P.
1.	2.	3.	4.	5.	6.	7.	8.
1 : 1	1.000	45.	80.5	1.414	45.	80.5	1.414
9 : 10	.900	41.98	76.0	1.486	48.02	84.5	1.337
8 : 9	.888	41.63	75.6	1.495	48.37	85.0	1.329
7 : 8	.875	41.18	75.0	1.504	48.82	85.5	1.317
6 : 7	.857	40.60	74.1	1.518	49.40	86.3	1.302
5 : 6	.833	39.80	73.0	1.536	50.20	87.4	1.280
4 : 5	.800	38.66	71.1	1.562	51.34	88.8	1.249
7 : 9	.777	37.85	70.0	1.579	52.15	89.6	1.228
3 : 4	.750	36.83	68.5	1.600	53.17	90.8	1.200
5 : 7	.714	35.53	66.2	1.628	54.47	92.5	1.162
7 : 10	.700	34.99	65.1	1.638	55.01	93.0	1.147
2 : 3	.666	33.68	63.2	1.664	56.32	94.5	1.109
5 : 8	.625	32.00	60.4	1.696	58.00	96.3	1.060
3 : 5	.600	30.96	58.7	1.715	59.04	97.3	1.029
4 : 7	.571	29.75	56.6	1.736	60.25	98.5	.992
5 : 9	.555	29.05	55.4	1.748	60.95	99.1	.971
1 : 2	.500	26.56	51.0	1.789	63.44	101.4	.894
4 : 9	.444	23.94	46.3	1.827	66.06	103.6	.812

BEVEL GEAR TABLE FOR SHAFTS AT 90°.

PROPORTION OF PINION TO GEAR.		PINION.			GEAR.		
		A.	B.	2 E.	90° — A.	B.	2 F.
		Center Angle.	Angle Increment. Divide by n.	Diameter Increment. Divide by P.	Center Angle.	Angle Increment. Divide by N.	Diameter Increment. Divide by P.
1.	.2.	3.	4.	5.	6.	7.	8.
3 : 7	.428	23.20	45.	1.838	66.80	104.1	.788
2 : 5	.400	21.80	42.5	1.857	68.20	105.2	.743
3 : 8	.375	20.55	40.3	1.873	69.45	106.0	.702
1 : 3	.333	18.43	36.1	1.897	71.57	107.4	.632
3 : 10	.300	16.70	32.8	1.915	73.30	108.4	.575
2 : 7	.285	15.95	31.4	1.923	74.05	108.8	.549
1 : 4	.250	14.03	27.8	1.940	75.97	109.8	.485
2 : 9	.222	12.53	24.6	1.952	77.47	110.5	.434
1 : 5	.200	11.31	22.5	1.961	78.69	111.0	.392
2 : 11	.181	10.30	20.4	1.968	79.70	111.3	.357
1 : 6	.166	9.46	18.8	1.973	80.54	111.6	.329
2 : 13	.153	8.75	17.4	1.977	81.25	111.8	.304
1 : 7	.143	8.13	16.4	1.980	81.87	112.0	.283
2 : 15	.133	7.60	15.0	1.982	82.40	112.1	.265
1 : 8	.125	7.12	14.2	1.985	82.88	112.2	.248
2 : 17	.117	6.70	13.3	1.986	83.30	112.3	.233
1 : 9	.111	6.33	12.6	1.988	83.67	112.4	.221
1 : 10	.100	5.70	11.3	1.990	84.30	112.6	.200

$$P = \frac{N}{D'} = \frac{n}{d'};$$

$$P' = \frac{\pi}{P};$$

$$\tan A = \frac{d'}{D'} = \frac{n}{N};$$

$$\tan B = \frac{2 \sin A}{n};$$

$$\tan C = \frac{2.25 \sin A}{n},$$

$$2\ E = \frac{2}{P} \cos A,$$

$$2\ F = \frac{2}{P} \sin A.$$

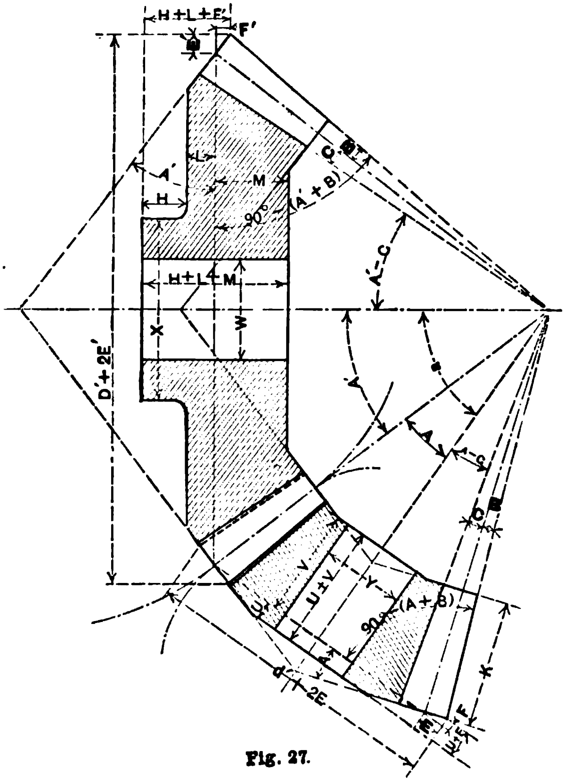


Fig. 27.

63. Bevel Gears with Axes at any Angle. If the axes of the gears intersect at angles other than 90°, the drawing of the blanks and development of the teeth do not differ from the cases already described. The figuring required is that indicated in Fig. 27, those in the heavy face being used to determine the other values, and not appearing on the finished drawing.

$$\tan A = \frac{\sin a}{\frac{N}{n} + \cos a} \cdot \quad \tan A' = \frac{\sin a}{\frac{n}{N} + \cos a};$$

$$\tan B = \frac{2 \sin A}{n} \text{ or } \frac{2 \sin A'}{N};$$

$$\tan C = \frac{2.25 \sin A}{n} \text{ or } \frac{2.25 \sin A'}{N}.$$

$$E = \frac{1}{P} \cos A; \quad E' = \frac{1}{P} \cos A'.$$

$$F = \frac{1}{P} \sin A; \quad F' = \frac{1}{P} \sin A'.$$

Or, the values for E, F, E', and F' may be obtained from the table for shafts at 90°, pages 54 and 55 by determining the center angles A and A', and finding the values for 2 E and 2 F, corresponding to each gear separately.

CHAPTER VII.

SPECIAL FORMS OF ODONTOIDS, NOTATION, FORMULAS, ETC.

64. Odontographs and Odontograph Tables. If tooth curves are to be drawn according to some established system, in which the angle of pressure is constant, or but one diameter of rolling circle be used, it may be desirable to employ some of the approximate methods for shortening the operation. While it is unnecessary for the student to familiarize himself with the theory, or even the details, of operating the various systems of approximating these curves, it is essential that a knowledge be had of the more useful tables and methods to which reference may be made when required.

Three methods are employed for approximating the odontoidal curves.

First, by circular arcs, the centers and radii of which are given in tables, or established by instruments, designed for this purpose.

Second, by curved templets from which the curves may be traced directly.

Third, by ordinates.

65. Willis's Odontograph. Among those of the first type, the oldest, best known, and least accurate, are the odontographs designed by Professor Willis. When used for gears having a large number of teeth, the error is very slight; but in the case of involute teeth of small number it is very noticeable. Fig. 28 illustrates the application of this instrument to the drawing of curves of the cycloidal system. The centers for the circular arcs designed to approximate the curves are found on the straight edge, A B, and at a distance from the zero point of the scale to be found in the published table accompanying the instrument.

The theory and application of these odontographs is clearly treated of in the instructions accompanying these instruments, also in Stahl and Wood's "Elements of Mechanism," pages 113 to 122, and more briefly in MacCord's "Kinematics," pages 172 to 174.

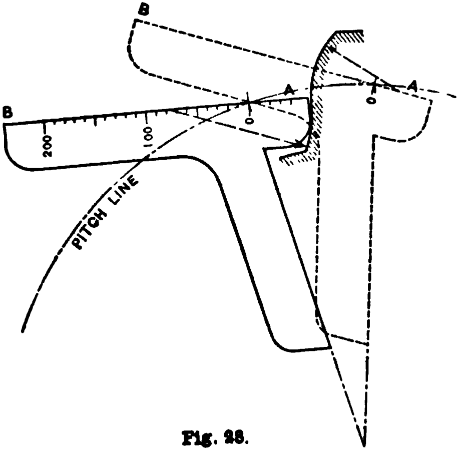


Fig. 28.

66. The "Three Point Odontograph," designed by Mr. Geo. B. Grant, is a table for face and flank radii and centers, figured for circular arcs passing through the three most important points of the tooth curves; viz., at addendum or dedendum circles, pitch circle, and a point midway between. This gives a very close approximation to the true curve for the system which has radial flanks for gears of twelve teeth. The tables and instructions are published in Grant's "Odontics," pages 41 and 42, and in Stahl and Wood's "Elements of Mechanism," pages 124 and 125.

67. The Grant Involute Odontograph, designed by Geo. B. Grant, and published in his "Odontics," pages 29 and 30, gives a very close approximation to the involute for 15° angle of pressure and epicycloidal extension, all gears being designed to engage a 12-toothed gear without interference.

68. The Robinson Odontograph differs from the preceding in that it is an instrument having a curved edge which is used as a templet to trace the tooth curve, tables being used to determine the position of the instrument with relation to the pitch circle.

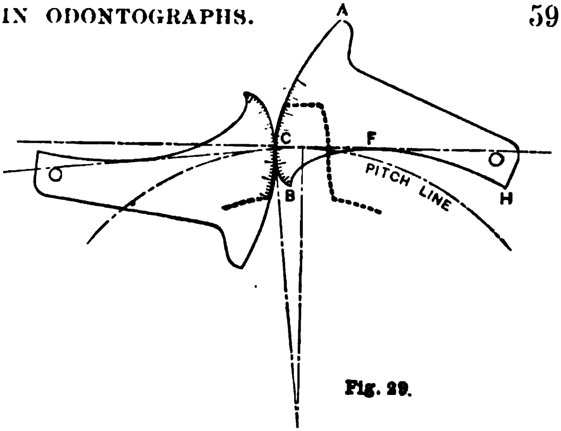


Fig. 29.

Fig. 29 illustrates the instrument in position. The curve B C A is a logarithmic spiral, and the curve B F H the evolute of the first, and therefore a similar and equal spiral. By means of this instrument, in connection with the published tables accompanying it, involute teeth may be drawn as well as cycloidal, and a much larger range of the latter is possible than is afforded by the Willis odontograph. The theory of this instrument is best treated by Professor Robinson in Van Nostrand's *Eclectic Magazine* for July, 1876, and Van Nostrand's "Science Series," No. 24. Also see Stahl and Wood's "Elements of Mechanism," pages 126 to 130.

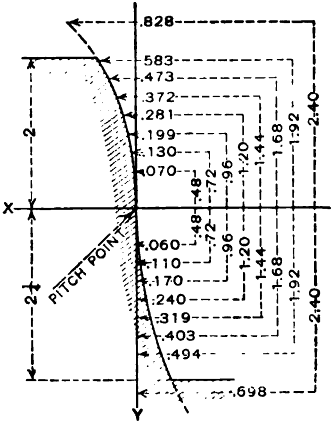


Fig. 30.

69. The Klein Coördinate Odontograph. Fig. 30 is designed to eliminate the labor of drawing pitch circles of large radii by constructing the curve by ordinates from a radial line. The tables and explanation of the method may be

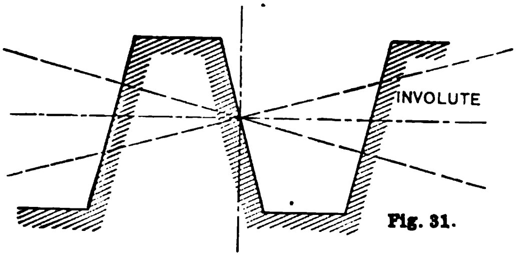


Fig. 31.

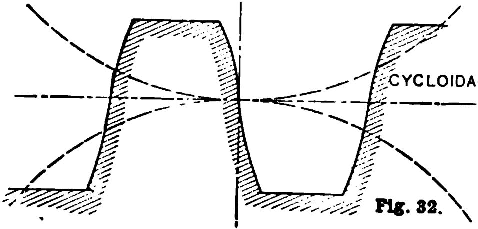


Fig. 32.

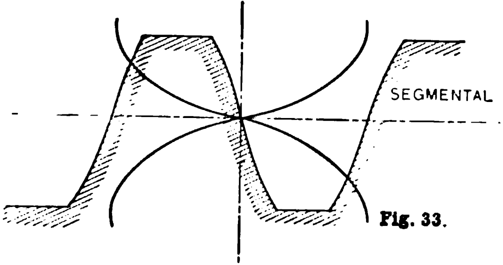


Fig. 33.

found in Professor Klein's "Elements of Machine Design," page 50.

70. Special Forms of Odontoids and their Lines of Action. Gears may be classified from the forms of rack teeth, as follows:

System.	Tooth Curve.	Line of Action.
Involute, Fig. 31,	A right line,	A right line.
Cycloidal, Fig. 32,	A cycloid,	A circular arc.
Segmental, Fig. 33,	A circular arc,	Conchoid of Nicomedes.

In like manner other systems might be derived from, and classified by, the forms of their rack teeth.

It is of interest to note in connection with the first two that any tooth of either system may be derived from a right line. In the cycloidal system the addendum of any gear tooth will properly engage the radial flank of some gear. If, therefore, the addenda of any gear tooth be made to fit the dedenda of teeth consisting of radial flanks, the resulting teeth must be cycloidal. A skilled mechanic with file and straight-edge could in this manner produce the templet for any desired cycloidal tooth without the aid of other mechanism. Of course such a method would require considerable skill in producing a perfect tooth, and it is not the best means to the end; but it is of much interest to the student as illustrating the relation between the mechanical and graphic methods of attaining the same end. In like manner we may produce templets for involute teeth from the right line rack tooth of the system.

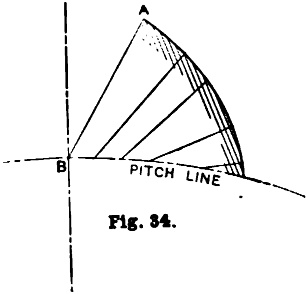


Fig. 34.

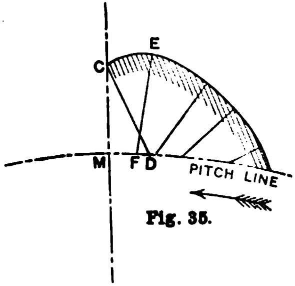


Fig. 35.

71. Conjugate Curves. — The curves of any pair of teeth being so related as to produce a uniform velocity ratio are called conjugate, or odontoids, and if any tooth curve of reasonable form be assumed, a second curve may be obtained which shall be conjugate to the first. By a reasonable form is meant the conformity to the following principle: —

The normals to the curve must come into action consecutively, as in Fig. 34, and not as in Fig. 35, in which it will be seen that the normal E F will pass through the pitch point M, and the point E come into

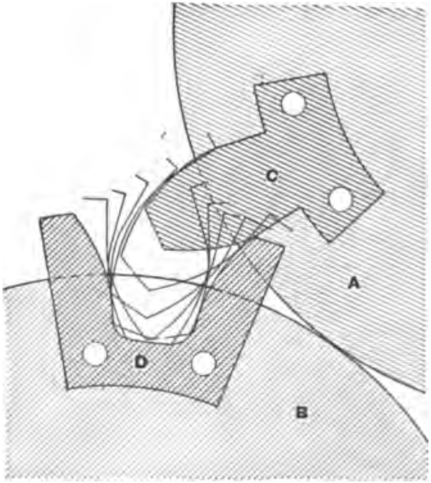


Fig. 36.

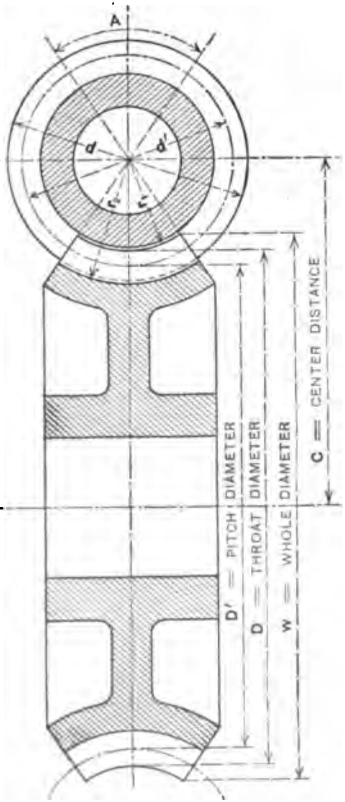


Fig. 37.

action before the point C, which is impossible. Let C, Fig. 36, be any tooth form conforming to the above condition, and the periphery of disk A its pitch line. Suppose it is required to derive its conjugate having for its pitch circle the periphery of disk B. This may be obtained by a graphic process, as in Art. 28, page 15, or by the mechanical method known as the molding process of Fig. 36. C is a templet of the given tooth form, which is fastened to disk A, and revolving in contact with disk B, the disks maintaining a constant velocity ratio. The successive positions of C are then traced on the plane of disk B, and the tangent curve will be that of the required conjugate tooth.

The method is applicable to all forms of spur gear teeth, but to only one form of bevel gear, the octoid.

72. Worm Gearing. A worm is a screw designed to operate a gear, called a worm wheel or gear, the axis of the latter being perpendicular to that of the worm. Art. 3, page 3. The section of a worm and gear made by a plane perpendicular to the axis of the gear, and including the axis of the worm, is identical with that of a rack and gear of the same system and pitch. The worm, or screw, may be single, double, etc. If single, the circular pitch corresponds with the pitch of the thread; if double, the circular pitch will be half the pitch of the thread, etc. To avoid misunderstanding, it is customary to speak of the pitch of the thread as the lead.

A drawing of the tooth form is required only in special cases of large cast gears, and the usual representation is that shown by Fig. 37.

The diameter of the worm is commonly made equal to four or five times the circular pitch, and the angle A varies from 60° to 90°.

FORMULAS FOR WORM AND GEAR.

L = Lead of worm;
m = Threads per inch in worm;
d = Outside diameter of worm;
d' = Pitch diameter of worm;
W = Whole diameter of gear;
D = Throat diameter of gear;
D' = Pitch diameter of gear;
$L = \frac{1}{m} = P'$, for single threads,
$L = \frac{2}{m} = 2\,P'$, for double threads, etc.;
r_1 and r_2 are dimensions required for the hob, or cutter, employed in cutting the worm gear;
C = Center distance;

$$P' = \frac{\pi\,D}{N+2};$$

$$D' = \frac{N\,P'}{\pi} = \frac{N}{P};$$

$$D = \frac{N+2}{P};$$

$$r_1 = \frac{d}{2} - \frac{2}{P};$$

$$r_2 = r_1 + \frac{9}{8}\,P;$$

$$C = \frac{D+d}{2} - \frac{1}{P};$$

$$W = D + 2\left(r_1 - r_1 \cos\frac{A}{2}\right).$$

73. Literature. The following list of books and articles is published to assist the student who may wish to pursue the subject beyond its elementary stage. Only those treatises have been enumerated which are likely to be accessible and useful. The great works of Willis, Rankine, and Reuleux are omitted, as the student will derive more benefit from the interpretation of these works by later authors than by a study of the original treatises.

"The Mechanics of the Machinery of Transmission," revised by Professor Herrmann, is Vol. III., Part I., Sect. 1, of Weisbach's "Mechanics of Engineering." This work includes one of the most valuable treatises on the subject of gearing, but it is somewhat difficult. Wiley, $5.00.

"Kinematics," by Professor MacCord, is chiefly devoted to the subject of gearing. It contains much original matter of importance. No student of the subject can afford to do without this treatise. Wiley, $5.00.

"Elements of Machine Design," by Professor Klein, was published for the students of Lehigh University. Several chapters are devoted to gearing, and include some excellent tables and problems. The Klein coördinate odontograph is fully illustrated and explained. J. F. Klein, Bethlehem, Pa., $6.00.

"Odontics," by Mr. Geo. B. Grant, is one of the most valuable modern treatises on gearing. It is both theoretical and practical. It is concise, contains many useful tables, and is well illustrated. The subject cannot be pursued to advantage without its use. Lexington Gear Works, Lexington, Mass., $1.00.

"Practical Treatise on Gearing," by Mr. O. J. Beale. An excellent practical treatment of the design and construction of gears. It deals little with the theory, but that little is thoroughly and simply taught. Brown & Sharpe Manufacturing Company, Providence, $1.00.

"Formulas in Gearing." This is published by the Brown & Sharpe Manufacturing Company, and contains many useful formulas for the draftsman, and valuable hints for the cutting of gears. $2.00.

"Elementary Mechanism," by Professors Stahl and Wood, is a most comprehensive text book on the subject of gearing. It is well classified, contains numerous examples, and is a valuable reference book for the student. Van Nostrand, \$2.00.

The student is recommended to read the following articles published in the *American Machinist*.

"Cutting Bevel Gears in a Universal Milling Machine," by O. J. Beale, June 20, 1895.
"Planed Bevel Gear Teeth," by George B. Grant, Dec. 9, 1896.
"Grant's Epicycloidal Bevel Gear Generator," June 7, 1894.
"Bilgram Bevel Gear Cutting Machine," May 9, 1885.
"Bilgram Gear Exhibit," Oct. 12, 1893.
"Bevel Gear Curves." Chart for plotting from Grant's bevel gear chart, by H. Walden, Oct. 8 1896 (chart corrected Nov. 5, 1896).
"The Strength of Gear Teeth," by Henry Hess, Feb. 18, 1897.
"Strength of Gear Teeth," by W. T. Sears, June 10, 1897.
"Gear Arm Proportions," by Henry Hess, April 29, 1897.

74. Notation and Formulas.

SPUR GEARS.

P' = Circular pitch, ART. 17, page 11;
P = Diameter pitch, ART. 18, page 11;
D' = Pitch diameter of gear;
D = Whole, or addendum, diameter of gear;
d' = Pitch diameter of pinion;
d = Whole, or addendum, diameter of pinion;

N = Number of teeth in gear;
n = Number of teeth in pinion;
s = Addendum of tooth, ART. 31, page 17;
f = Clearance, ART. 27, page 15;
t = Thickness, ART. 31, page 17;
p = Least angle of pressure, ART. 45, page 33;

$\pi = 3.1416$;

$P' = \frac{\pi D'}{N} \cdot \frac{N}{D'} = \frac{\pi}{P'}$, **Art. 17, page 11;**

$P = \frac{N}{D'}$, $P\,P' = \pi$, **Art. 18, page 12;**

$s = \frac{1}{P}$, $f = \frac{s}{8} = \frac{1}{8\,P}$, **Art. 31, page 17;**

$t = \frac{P'}{2} = \frac{\pi}{2\,P} = \frac{1.57}{P}$, **Art. 31, page 17;**

$D = D' + 2\,S = D' + \frac{2}{P} = \frac{P\,D' + 2}{P} = \frac{N+2}{P}$;

$\cos p = \sqrt{\frac{N-2}{N}}$, **Art. 45, page 33.**

ANNULAR GEARS.

$$\left.\begin{array}{ll} Rg = & \text{Radius of gear,} \\ Rp = & \text{Radius of pinion,} \\ r_1 = & \text{Inner describing circle,} \\ r_2 = & \text{Outer describing circle,} \\ r_3 = & \text{Intermediate describing circle,} \\ C = & \text{Center distance,} \end{array}\right\} \text{ Art. 50, page 39;}$$

$Rg = r_3 + r_1$, **Art. 50, page 39;**

$Rp = r_3 - r_2$, **Art. 50, page 39;**

$C = Rg - Rp$, **Art. 50, page 39;**

r_1 maximum $= Rg - C$; r_1 minimum $= 0$, **Art. 52, page 40;**

r_2 maximum $= C$; r_2 minimum $= 2\,C - Rg$, **Art. 52, page 40;**

r_3 maximum $= Rg$; r_3 minimum $= C$, **Art. 51, page 40.**

BEVEL GEARS, SHAFTS AT 90°, ART. 61, PAGE 56.

A = Center angle of pinion;

B = Angle increment;

C = Angle decrement;

E = One-half the diameter increment for pinion;

F = One-half the diameter increment for gear.

$\tan A = \frac{d'}{D'} = \frac{n}{N}$;

$\tan B = \frac{2 \sin A}{n}$;

$\tan C = \frac{2.25 \sin A}{n}$;

$E = \frac{1}{P} \cos A$ $\qquad F = \frac{1}{P} \sin A$.

BEVEL GEARS, SHAFTS AT OTHER THAN 90°, ART. 63, PAGE 56.

a = Angle of shafts;

A = Center angle of pinion;

A′ = Center angle of gear;

B = Angle increment;

C = Angle decrement;

E = One-half the diameter increment for pinion;

E′ = One-half the diameter increment for gear;

F = Dimension required for backing of pinion;

F′ = Dimension required for backing of gear.

$$\tan A = \frac{\sin a}{\frac{N}{n} + \cos a};$$

$$\tan A' = \frac{\sin a}{\frac{n}{N} + \cos a};$$

$$\tan B = \frac{2 \sin A}{n} = \frac{2 \sin A'}{N};$$

$$\tan C = \frac{22.5 \sin A}{n} = \frac{2.25 \sin A'}{N};$$

$$E = \frac{1}{P} \cos A \qquad E' = \frac{1}{P} \cos A';$$

$$F = \frac{1}{P} \sin A \qquad F' = \frac{1}{P} \sin A';$$

WORM GEARS, ART. 72, PAGE 62.

L = Lead of worm;

m = Threads per inch in worm;

d = Outside diameter of worm;

d′ = Pitch diameter of worm;

D = Thread diameter of gear;

D′ = Pitch diameter of gear;

W = Whole diameter of gear.

$$L = \frac{1}{m} - P' \text{ for single threads;}$$

$$L = \frac{2}{m} = 2P' \text{ for double thread, etc.,}$$

$$D' = \frac{N}{P} \quad \cdot \quad D = \frac{N + 2}{P};$$

$$P' = \frac{\pi D}{N + 2} \quad \cdot \quad W = D + 2\left(r_1 - r_1 \cos \frac{A}{2}\right);$$

$$C = \frac{D + d}{2} - \frac{1}{P};$$

$$r_1 = \frac{d}{2} - \frac{2}{P} \quad \cdot \quad r_2 = r_1 + \frac{9}{8} P.$$

CHAPTER VIII.

PROBLEMS.

75. Method to be Observed in Performing the Problems. No attempt should be made to graphically solve the following problems until the general principles involved are well understood.

The first requisite to this is the mastery of Chapter II., on Odontoidal Curves; and this can be best acquired by the drawing of the various curves, together with a study of their characteristics. No problems have been given on this topic, but the following course of study would be desirable: —

Having prescribed diameters for rolling circles and director, or pitch circles, draw a cycloid. epicycloid, and hypocycloid, as described in ARTS. 5, 6, and 7, page 5. Obtain a sufficient number of points in each case to enable the curves to be drawn free-hand with considerable accuracy, after which they may be corrected by the use of scrolls. Next prescribe a point on each (not one already found), and draw normals to each by ART. 8, page 5.

The second method, ART. 9, page 5, is the more practical, and should also be studied by drawing a small part of each curve, beginning at a point on the director circle.

It is also desirable that one of the epitrochoidal forms be drawn, and a normal determined. ART. 11, page 6.

The problems are designed to be solved on a sheet which shall measure 10″ by 14″ within the margin line, and the lay-out of these sheets is given on PLATES 14 and 15, there being four problems on each plate. Measurements are from the margin line.

It is unnecessary to represent all the teeth in a gear, but such as are shown should be drawn with the greatest accuracy attainable by the student. Without this care the study will avail one little, and the time consumed in discovering errors will be great.

The inking of the curves may be omitted if time will not admit of its being well done; but in either case it is desirable to emphasize the curves, and distinguish clearly between the gears by making a *very* light wash of color on the inside of the curve, the width to be about one-quarter of an inch. One color may be used for the pinion, and a second for the rack and gear.

Problem 1, Plate 14, Fig. 1. Cycloidal Limiting Case. Face or Flank only.

EXAMPLE.	D′	d′	N	n	A	B
1	10		15	12	3¾	4½
2	10	8		12	3½	4½
3	12¼	7	21		3	4
4		7½	15	10	3½	4¼
5	10	8	15		4¼	4½
6	10½		14	10	3¼	4½
7		6	24	12	2¾	4
8	12¼		21	12	3¼	4½

STATEMENT OF PROBLEM. Having given the diameters of pitch circles, number of teeth, and diameter of describing circle, it is required to draw the teeth for pinion, gear, and rack, having arcs of contact equal to the pitch, and contact on one side of pitch point only.

Study ARTS. 1 to 26 before performing this problem.

OPERATIONS. 1. By ART. 18, page 11, determine the value of N, n, D' or d, one of which is omitted from the table. Observe that $\frac{D'}{d'} = \frac{N}{n}$.

2. Draw center and pitch lines and describing circle. Lay off the circular pitch on each gear by spacing the circumferences into as many parts as there are teeth.

3. Obtain the first point of contact by laying off from the pitch point on the describing circle an arc equal to the circular pitch, the direction being determined by the rotation required. ART. 16, page 10. ART. 21, page 12. ARTS. 22 and 23, page 13.

4. With the above describing point, generate the face and flank required. ARTS. 14 and 15, page 10.

5. Draw the working faces of gear teeth, and assuming the gear teeth to be pointed, draw opposite side of each. ART. 16, page 10.

6. Draw the working flanks of the pinion teeth, observing that the depth must be sufficient to admit the gear teeth, but without clearance. Obtain the thickness, and draw the opposite sides. ART. 16, page 10.

7. Draw the describing circle for rack. Obtain the first point of contact between pinion and rack, and describe the cycloid for rack teeth. Construct rack teeth. ART. 25, page 14. Note that thickness of rack tooth must equal space between pinion teeth, or thickness of gear teeth, measured on the pitch line.

8. To determine points of contact of conjugate teeth, assume any point on face of gear tooth, and determine, first, its position when in contact with the pinion; second, the point of the pinion tooth engaging it. Since the contact must take place on the path of contact, ART. 21, page 12, the assumed point will lie at the intersection of this arc and one described

through the given point from center of gear. To solve the second, describe an arc from the center of the pinion through the point previously determined, and its intersection with the pinion flank will be the engaging point required.

Next construct the normals for each of these points. ART. 8, page 5. They should be equal to each other, and also to the distance from the pitch point to the point on the path of contact in which they engage. ART. 14, page 10.

9. Obtain the maximum angle of obliquity, or pressure, between gear and pinion, pinion and rack. ART. 24, page 14.

Problem 2, Plate 14, Fig. 2. Cycloidal Limiting Case. Face and Flank. Study ARTS. 26 to 30.

STATEMENT OF PROBLEM. The diameters of gears, number of teeth, and describing circles being given, it is required to draw the teeth for pinion, gear, and rack, when the arc of approach = the arc of recess = half the circular pitch, the flank of gear being radial.

OPERATIONS. 1. Draw center lines, pitch lines, and rolling circles, the second circle being determined by ART. 9, page 6. Divide the pitch circle into the required parts to obtain the circular pitch.

2. Lay off arcs equal to $\frac{P'}{2}$ on each of the rolling circles to obtain the first and last points of contact, observing the direction of rotation prescribed in Fig. 2.

3. With the point thus determined on small rolling circle, describe the addendum of gear tooth and dedendum of pinion tooth. With the point on the second describing circle generate the addendum of pinion tooth. The dedendum of gear tooth being radial may then be drawn. Make the dedenda of pinion and gear deep enough to admit the engaging addenda, but allow no clearance.

4. Draw the working faces of the pinion teeth and then the opposite faces to make the teeth pointed. Similarly draw the gear teeth, making them pointed also. The sum of the thickness of the teeth cannot be greater that the circular pitch. Art. 29, page 16. In this case it will be found to be about one-hundredth of an inch less, which will be the backlash. An increase in the diameter of either rolling circle would make the solution impossible.

5. Draw the dedenda of pinion and gear teeth.

6. The describing circles for the rack teeth will be determined by Art. 14, page 10. Draw the circles with their centers on the line of centers, and obtain the first and last points of contact. These points should fall on the addendum and dedendum of pinion teeth already drawn, as in Plate 5 at M and O. From these points describe the addenda and dendenda of the rack teeth. The thickness of these teeth must equal those of the gear.

7. Obtain the maximum angle of pressure for approach and recess between pinion and gear and pinion and rack. It would also be desirable to obtain the curve of least clearance in one case. Art. 28, page 15.

Problem 3, Plate 14, Fig. 3. Cycloidal Gear. Practical Case. Complete Chapter III. before performing this problem.

Example	d'	N	n	A	a	C
1	9	18	12	4	5	3½
2	8	21	12	4	4	3⅞
3	10	20	16	3½	4	3
4	8	22	12	3½	5	4
5	9	16	12	4	5	3½
6	7	20	12	3	4	4½

Statement of Problem. The diameters of pitch circles and rolling circles being given, and the number of teeth known, it is required to draw the teeth for gear, pinion, and rack, to obtain the maximum angle of obliquity, and the arcs of approach and recess in each case. The teeth will be standard with $\frac{1}{32}''$ backlash. Art. 31, page 17. Art. 71, page 61.

Operations. 1. Figure the diameter of gear, circular, and diametral pitch, Arts. 17 and 18, page 11, and determine proportions of teeth. Art. 31, page 17.

2. Draw center lines, pitch lines, addendum, and dedendum circles, and rolling circles. Divide the pitch circle into as many parts as there are teeth, beginning to space at the pitch point.

3. Beginning at the pitch point, describe pinion flank, gear face, gear flank, and pinion face, by Art. 9, page 5. See also Art. 34, page 21.

4. Lay off thickness of teeth, Art. 31, page 17, and describe addenda of pinion and gear teeth by approximate method. Art. 34, page 22. Describe dedenda by Art. 16, page 11. Draw fillets. Art. 31, page 18.

5. Describe rack teeth.

6. Determine the following for gear, pinion, and rack in terms of P′. Arts. 21 to 24 inclusive, pages 12, 13, and 14, Art. 32, page 18.

	Pinion and Gear.	Pinion and Rack.
Arc of approach		
Arc of recess		
Arc of contact		
Maximum angle of pressure		

Problem 4, Plate 14, Fig. 4. Involute Limiting Case. Study ARTS. 38 to 42.

STATEMENT OF PROBLEM. Number of teeth five and six. Pinion teeth pointed. No backlash or clearance. Arc of contact equal to the circular pitch.

This problem being similar to that of PLATE 8, reference will be made to that figure.

The case being a limiting one, the distance between the points of tangency of base circles and line of pressure must equal one-sixth of the circumference of the gear base circle, or one-fifth of the circumference of the pinion base circle. The tangent of the angle of pressure will equal $\frac{AB}{AF} = \frac{BD}{DG} = \frac{AD}{AF + DC}$, but $AD = DKC$ by construction, and $DKC = \pi$. Also $AF + DC = 5\frac{1}{2}$, hence, $\frac{AD}{AF + DC} = \frac{\pi}{5.5}$ = tan. of the angle of pressure. The angle corresponding to this tangent is 29° 44′ 6″. The distance between the centers will be $\sqrt{AD^2 + \overline{AF + DG}^2} = \sqrt{\pi^2 + 5.5^2} = 6\frac{1}{3}$.

The angle of pressure and distance between centers could have been determined graphically by laying off FA, in any direction, equal to the radius of pinion base circle, AD perpendicular to FA, and equal to one-fifth of pinion base circle. Finally, DG perpendicular to AD, and equal to the radius of gear base circle.

OPERATIONS. 1. Draw the line of centers, base circles, and line of pressure. Determine the points of tangency, which limit the action in either direction, and through the pitch point, determined by the intersection of the line of centers and line of pressure, draw the pitch circles. It is desirable now to test AD by proving it equal to one-fifth of the pinion base circle, or one-sixth of the gear base circle.

2. Draw the involute AC, PLATE 8, of the gear, and DP of the pinion. ART. 12, page 7. ART. 38, page 26. Determine the circular pitch, and lay off as many divisions as there are teeth to be drawn. Copy the curves already drawn.

3. Draw the opposite face of pinion teeth, making them pointed. To draw the opposite faces of gear teeth proceed as follows: Since contact between the opposite faces must take place along the line of action C E, PLATE 8, the contact between the engaging teeth will be at E. At E draw arc E 1 from center G. Bisect this arc, and lay off M and H from this radial bisector equidistant with A and C. Through these points describe the curve of opposite face, and draw the remaining teeth.

That portion of the teeth lying within the base circle will be radial, and extend sufficiently to admit the engaging teeth, but without clearance.

4. Construct two rack teeth. ART. 40, page 29.

5. Epicycloidally extend the gear teeth so as to make them pointed. Similarly extend the rack teeth, but only as much as the clearance for the pointed gear tooth will permit. ART. 41, page 30.

Problem 5, Plate 15, Fig. 1. Involute Practical Cases. Complete the study of Chapter IV.

STATEMENT OF PROBLEMS. Several gears and racks are given to describe involute teeth of standard dimensions. To determine the interference, if there be any, and to correct the curves for the same.

OPERATIONS. 1. Draw three or four teeth of gear A, and two teeth of engaging pinion B, the angle of pressure being 15°. ART. 42, page 32, Fig. 16. Make contact at pitch point in all cases. Correct for interference by epicycloidal extension. ART. 31, page 17. ART. 42, page 30. ART. 41, page 32.

2. Draw three or four teeth of gear A engaging rack F.

3. Draw three teeth of pinion B engaging rack E, and correct rack teeth for interference.

4. Draw a portion of gear C and rack K, the angle of pressure being 20°. Test this for interference by ART. 45, page 33, as well as by graphic method.

5. Draw a few teeth of gear D, the angle of pressure being 15°. Determine the least number of teeth that will engage it without interference.

Problem 6, Plate 15, Fig. 2. Cycloidal Annular Gear. Study ARTS. 48 to 56.

EXAMPLE.	D′	d′	N	n	A	a	B
1	19½	9	13	6	7	3½	5½
2	19½	9	13	6	6½	4	5½
3	19½	9	13	6	6	4½	5½
4	17½	7	15	6	7	3½	5½
5	17½	7	15	6	7½	3	5½

STATEMENT OF PROBLEM. The number of teeth and diameters of pitch and describing circles being given, it is required to draw the tooth outlines, and determine the increased arc of contact due to secondary action. The arc of contact, not including that due to the secondary action, is equal to the circular pitch, and the arc of approach equals the arc of recess.

OPERATIONS. 1. Draw the center and pitch lines and describing circles.

2. Determine the circular pitch, and lay off half this amount from the pitch point on each of the describing circles to determine the first and last points of contact.

3. Describe the curves of the teeth.

4. Determine the intermediate describing curve, and draw the same to obtain the limit of secondary action.

5. Determine the maximum angle of pressure for approach and recess. Also the angle of pressure for the last point of secondary action, and the increase in the arc of contact.

Problem 7, Plate 15, Fig. 2. Involute Annular Gear. Complete Chapter V.

EXAMPLE.	D′	d′	N	n	Angle of Pressure.	B
1	15	7½	20	10	20°	6½
2	15	6	30	12	15°	7
3	16	8	16	8	20°	6
4	20	8	30	12	15°	7
5	24	18	24	18	20°	3½

STATEMENT OF PROBLEM. The pitch diameters, number of teeth, and angle of pressure being given, it is required to draw the tooth curve, to determine if there will be any interference when the addenda of pinion teeth are made standard, and finally the length of the arc of contact in terms of P′.

OPERATIONS. 1. Draw center and pitch lines, line of pressure, and base circles.

2. Make addenda of pinion standard if a second engagement does not take place. ART. 56, page 43, and limit addenda of gear by ART. 56, page 43.

3. Determine the arc of contact in terms of P′.

Problem 8, Plate 15, Fig. 3. Cycloidal and Involute Bevel Gears. Shafts at 90°. Study ARTS. 57 to 63.

EXAMPLE.	P	N	n	Q	K	L	M	H	W	X	U	V	Y
1	3	18	15	3½	1½	⅝	⅝	¾	1½	3	⅝	⅞	1½
2	4	24	20	3½	1¼	½	½	¾	1⅜	2½	¾	¾	1⅛
3	2	16	12	3½	1¾	⅞	⅝	1	2	4	⅞	1⅛	1¾
4	4	28	20	3	1½	½	½	1	1⅜	2½	½	1	1⅛
5	3	21	15	3	1½	⅝	½	⅞	1½	2⅝	¾	1	1¼
6	2	14	12	3½	1¾	¾	¾	¾	1¾	3½	¾	1	1½
7	3	21	18	3⅜	1½	⅝	⅝	1	1¾	3½	⅝	⅞	1½
8	2	18	14	4	1⅞	¾	½	¾	2	4	¾	1	1¾
9	4	20	16	3½	1	⅜	½	⅝	1¼	2¼	½	¾	1
10	3	21	18	3⅜	1½	½	¾	1	1½	3¼	⅝	⅞	1½

If involute, make angle of pressure 15°.

If cycloidal, make diameter of rolling circles equal to the elements of normal cone of pinion.

STATEMENT OF PROBLEM. The proportions of the gear being given by the table, it is required to draw the gear blanks, describe the development of the teeth on the normal cones, and figure the gears.

OPERATIONS. 1. Having determined the pitch diameters, draw the gear blanks. ART. 60, page 48.

2. Describe two or three teeth of each gear on the developed surfaces of the outer and inner normal cones. ART. 60, page 48.

3. Figure the gears, ART. 61, page 51.

Problem 9, Plate 15, Fig. 4. Cycloidal and Involute Bevel Gears. Shafts at other than 90°. Study ART. 63.

EXAMPLE	a	P	N	n	Q	J	K	L	M	H	W	X	U	V	Y
1	40°	3	24	15	9	8⅞	2½	¼	1⅞	⅝	1½	3	¼	2¼	1¼
2	45°	3	24	15	9	7⅝	2¼	¼	1⅞	⅝	1½	3	¼	2	1¼
3	50°	4	34	24	8¾	8	2	¼	1½	¾	1½	3	¼	1¾	1¼
4	55°	3	27	21	9	8	2¼	⅜	1⅝	½	1¾	3¼	¼	1¾	1⅜
5	60°	2	20	12	8½	7¼	2½	⅝	1⅜	1	2	3½	⅜	2⅛	1⅝

If involute, make angle of pressure 15°.

If cycloidal, make diameter of rolling circles equal to the elements of normal cone of pinion.

STATEMENT OF PROBLEM. The proportions of the gear being given by the table, it is required to draw the gear blanks, describe the teeth on the development of the normal cones, and figure the gear.

OPERATIONS. 1. Determine the pitch diameters from above table, and draw the gear blanks.

2. Describe two or three teeth of each gear on the developed surfaces of the outer and inner normal cones.

3. Figure the gears.

INDEX.

Addendum defined, 12; proportion for, 17.
Angle decrement, 52.
Angle increment, 52.
Angle of edge, 51; of face, 51.
Angle of obliquity, or pressure, 14; affected by rolling circle, 18; constant, 28; for involute, 31; influence of, 33; method for determining, 33; reduced in annular gearing, 40.
Annular gear, notation, and formulas, 66; epicycloidal problem, 76; involute problem, 77.
Annular gearing, 38; secondary action in, 38; interchangeable with spur gearing, 42; involute system of, 43.
Approaching action detrimental, 17.
Approximate cycloidal curves, 22.
Approximation, Tredgold, 47; by circular arcs, 22
Arc of approach defined, 13.
Arc of contact defined, 13; relation to circular pitch, 16.
Arc of recess defined, 13.

Backing, 52.
Back cone, 48.
Backlash defined, 16; dimensions for, 18.
Base circle defined, 7, 27.
Base of system, 21; in annular gearing, 42.
Beale's "Practical Treatise on Gearing," 64.
Bevel gear defined, 2; Theory of, 45; character of curves employed, 46; drafting the, 48; blank, 49; length of face, 49; figuring the, 51; table for, 53, 54, 55; chart for plotting curves, 65; notation and formulas, 66; problems, 78, 79.
Bevel gears with axes at any angle, 56.
Bilgram, Hugo, inventor of octoid tooth, 47; machine for cutting bevel gear teeth, 47, 65; exhibit, 65
Brown & Sharpe publications, 64.

Circular pitch defined, 11.
Character of curves in bevel gearing, 46.
Clearance defined, 15; proportion for, 17.
Clock gears, 17.
Conchoid of Nicomedes, 60.
Conditions governing the practical case, 16.
Conjugate curves defined, 9, 61.
Constant angle of pressure, 28.
Constant velocity ratio defined, 1.
Conventional representation of spur gears, 25.
Contact, point of, 5; radius, 5; path of, 12; arc of, 13.
Coördinate odontograph, 59.
Crown gear, 47.
Curtate epitrochoid, 6.
Curve of least clearance, 15.

Curves, odontoidal, 4.
Cutting bevel gear teeth, 65.
Cutting angle, 51.
Cycloid, defined, 4; problem relating to, 68.
Cycloidal action, Theory of, 8.
Cycloidal curves, second method for describing, 5; approximated, 22.
Cycloidal system of annular gearing, 38.
Cycloidal annular gear problem, 76.
Cycloidal bevel gear problem, 78, 79.
Cycloidal limiting case problems, 69, 71.
Cycloidal practical case problem, 72.

Dedendum defined, 12; proportions for, 17.
Defects of involute system, 35.
Describing circle, defined, 4; a path of contact, 12; maximum and minimum, 16; influence on shape and efficiency of teeth, 18; relation to interchangeable gears, 20.
Describing disk, 8.
Describing point, 4.
Describing cone, 45.
Describing cylinder, 45.
Describing radius, 5.
Description of bevel gear table, 53.
Developed pitch circle, 50.
Development of normal cone, 49.
Diameter pitch, 11.
Director circle, 5.
Double contact in annular gearing, 39.
Double generation of epicycloid and hypocycloid, 6.
Drafting bevel gears, 48.

" Elements of Machine Design," 64.
" Elementary Mechanism," 65.
Epicycloid, defined, 5; second method for describing, 5; double generation, 6; spherical, 45; problem relating to, 68.
Epicycloidal extension, 30.
Epitrochoid, defined, 6; curtate, 6; prolate, 7; problem relating to, 68.
Exterior (outer) describing circle, 39; limitations of, 40, 41.

Face gearing, 2.
Face of gear, 24.
Face of tooth, 12.
Flank of tooth, 12; radial, 18.
Figuring bevel gears, 51.
Fillet, 18; size of, 18.
" Formulas in Gearing," 64.
Formulas for worm and gear, 63.
Formulas, Notation and, 65.

Gearing, 1.
Gear arm proportions, 65.
Gears, interchangeable, 20; face of, 24; comparison of, 24.
Generating point, 4.
Generating radius, 5.
Grant, Geo. B., bevel gear chart, 53; three point odontograph, 58; involute odontograph, 58; " Odontics," 58, 64; epicycloidal and bevel gear generator, 65.

Hyperboloid of revolution, 2.
Hyperbolic gears, 2.
Hypocycloid, defined, 5; second method for describing, 5; a radial line, 6; double generation, 6; spherical, 45; problem relating to, 68.

Influence of the angle of pressure, 33.
Influence of the diameter of rolling circle on shape and efficiency of teeth, 18.
Inner describing circle, 39; limitations of, 40, 41.
Inner normal cone, 48, 50.
Instantaneous radius, 4.
Intermediate describing circle, 39; limitations of, 40.
Internal gear, see annular gear.
Interference, 32; in annular gearing, 43.
Interchangeable gears, 20.
Involute, 4; defined, 7; system, 26; curves, character of, 27; rack, 28; system of annular gearing, 43; annular gear problem, 77; bevel gear tooth, 46; bevel gear problems, 78, 79; limiting case, 29; limiting case problem, 74; practical case, 30; practical case problem, 75.
Involute action, Theory of, 26; limit of, 28.
Involute gearing, defects of system, 35.
Involute teeth, epicycloidal extension of, 30.

"Kinematics," MacCord's, 64.
Klein's coördinate odontograph, 59; "Elements of Machine Design," 64.

Law of tooth contact, 10.
Lead of screw, 64.
Least angle of pressure, method for determining, 33.
Least number of teeth in annular gears, 42.
Limit of involute action, 28.
Limiting case, cycloidal, 10, 14; involute, 29; annular gearing, 38.
Limitations of intermediate, exterior, and interior describing circle, 40, 41.
Line of action a great circle, 47.
Literature, 63.
Logarithmic spiral, 59.

MacCord's "Kinematics," 64.
Method for determining least angle of pressure, 33.
Method to be observed in performing problems, 68.
"Mechanics of Engineering," 64.
"Mechanics of the Machinery of Transmission," 64.

Normal, defined, 4; to construct, 5; law governing, 61.
Normal cone, 48; development of, 49.
Notation and formulas, 65.

Obliquity, angle of, 14.
Octoid bevel tooth, 47, 62.
"Odontics," Grant's, 64.
Odontoid defined, 1; special forms of, 60.
Odontoidal curves, 4; problems relating to, 68.
Odontographs and odontograph tables, 57.
Odontograph, Willis, 57; Grant involute, 58; Grant Three point, 58; Robinson, 59; Klein, 59; coördinate, 59.
Outer describing circle, 39; limitations of, 40.
Outer normal cone, 48.

Path of contact defined, 12; affected by rolling circle, 18; a right line, 28.
Path of approach defined, 15.
Path of recess defined, 15.
Pitch cone, 48.
Pitch line, 10.
Pitch point, 9, 10, 27.
Pitch, circular, 11; diameter, 11.
Planed bevel gear teeth, 65.

Positive rotation defined, 11.
Practical case, conditions governing the, 16; cycloidal, 21; involute, 30; annular, 42.
"Practical Treatise on Gearing," 64.
Pressure, angle of, 14.
Prolate epitrochoid, 7.
Proportions for standard tooth, 17.
Problems, method to be observed in performing, 68.

Rack, 14; involute, 28; gears classified by, 60.
Radial flank, 18; as base of system, 21, 60.
Radius, describing, 5; contact, 5.
Rankine, 64.
Reuleux, 64.
Robinson odontograph, 59.
Rolling circle, see describing circle.
Rotation, positive, 1.

Screw gearing defined, 3.
Scroll, use of, 11.
Second method for describing cycloidal curves, 5.
Secondary action in annular gearing, 38, 41.
Segmental system, 60.
Skew gear defined, 3.
Spiral gear defined, 3.
Special forms of odontoids, 45.
Spherical epicycloid, 45.
Spherical hypocycloid, 45.
Spur gear defined, 2; illustrated, 10; having action on one side of pitch point, 10; having action on both sides of pitch point, 14; conventional representation, 25; interchangeable with annular gears, 42; notation and formulas, 65.

Theory of cycloidal action, 8.
Theory of involute action, 26.
Thickness of tooth, 17.
Three point odontograph, 58.
Tooth contact, law of, 10.
To construct a normal, 5.
Tredgold approximation, 47.

Unsymmetrical teeth, 37.
Use of bevel gear table, 53.

Velocity ratio constant, 1; not aeffcted by increase of center distance in involute, 28.

Weisbach's "Mechanics," 64.
Willis, odontograph of, 57; writings of, 64.
Worm gearing defined, 3, 62; notation and formulas for, 67.
Worm wheel, 62.

PLATE I.

Cycloid, Epicycloid, Hypocycloid and Involute curves.

REFERENCES TO TEXT.

ART. 4,	Page 4.	ART. 8,	Page 5.
5,	4.	9,	5.
6,	5.	12,	7.
7,	5.		

PLATE 1.

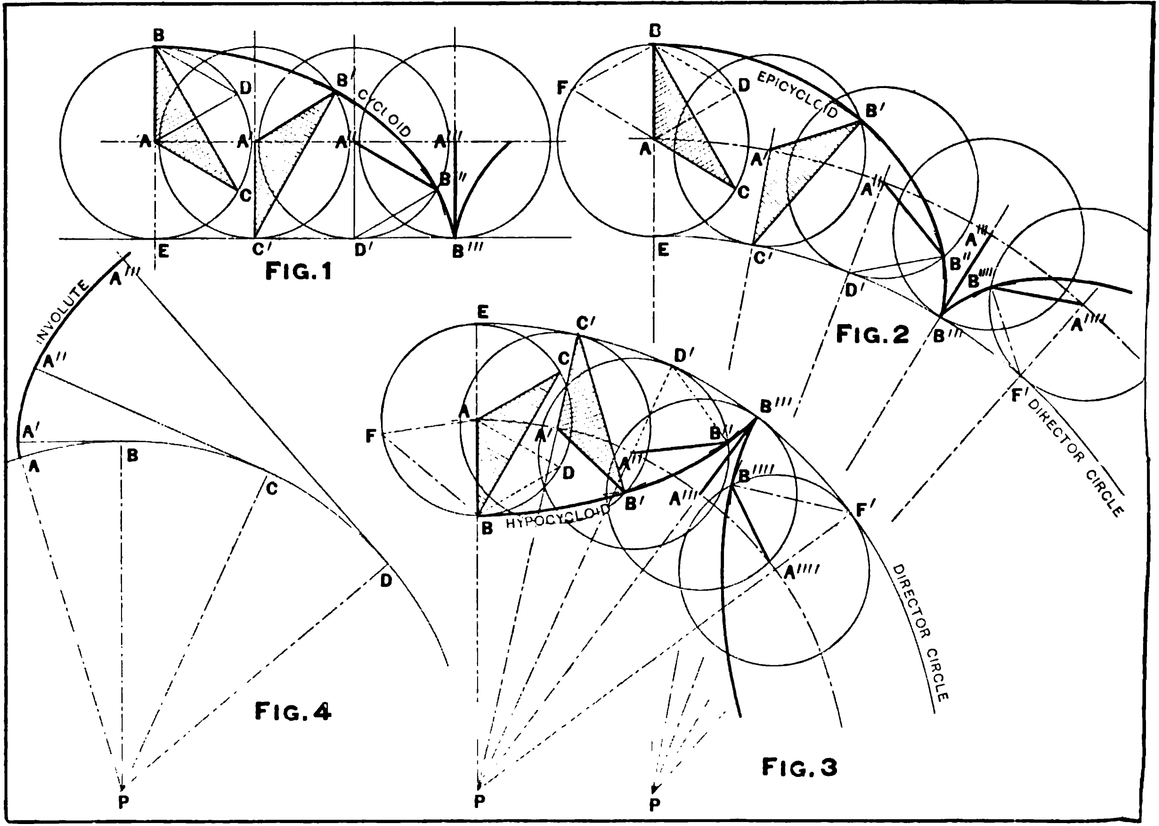


FIG. 1
B
D
B′
CYCLOID
A
A′
A″
A‴
B″
C
E
C′
D′
B‴
FIG. 2
B
F
D
EPICYCLOID
B′
A
A′
A″
C
E
C′
D′
A‴
B″
B⁗
B‴
A⁗
F′
DIRECTOR CIRCLE
FIG. 3
E
C′
C
D′
A
F
A′
D
A″
B″
B‴
B⁗
A‴
B′
B
HYPOCYCLOID
F′
A⁗
DIRECTOR CIRCLE
P
P
FIG. 4
INVOLUTE
A‴
A″
A′
A
B
C
D
P

PLATE 2.

Epitrochoidal curves. Double generation of Epicycloid and Hypocycloid. Approximate method.

REFERENCES TO TEXT.

ART.	10,	Page	6.
	11,		6.
	34,		22.

PLATE 2

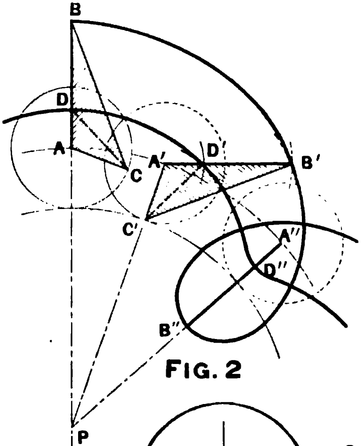


FIG. 2

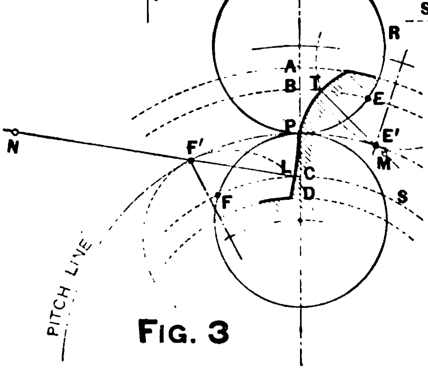


FIG. 3

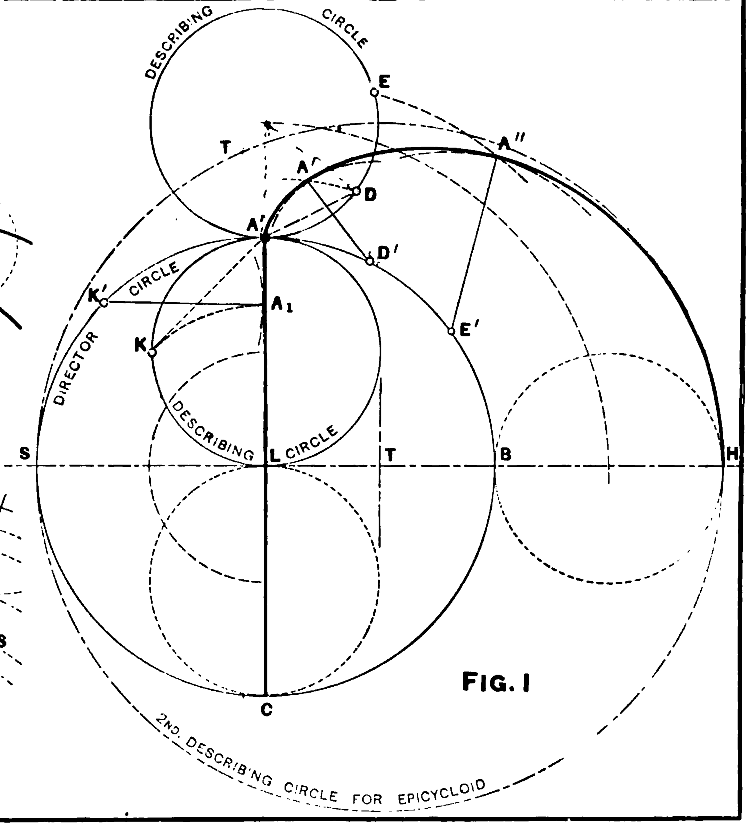


FIG. 1

PLATE 3.

Mechanical method for describing Odontoidal curves.

REFERENCES TO TEXT.

Art. 13,	Page 8.
15,	10.
21,	12.

PLATE 3.

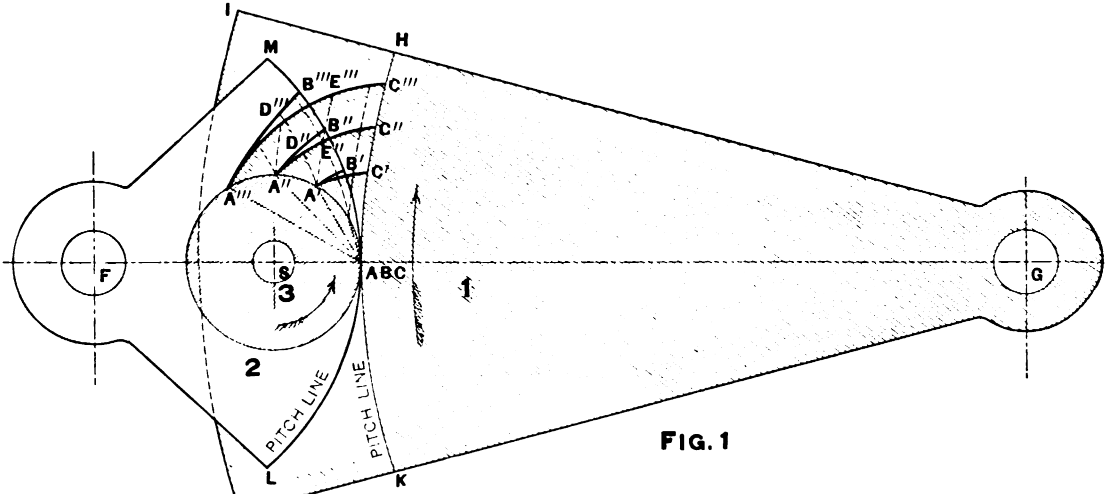


FIG. 1

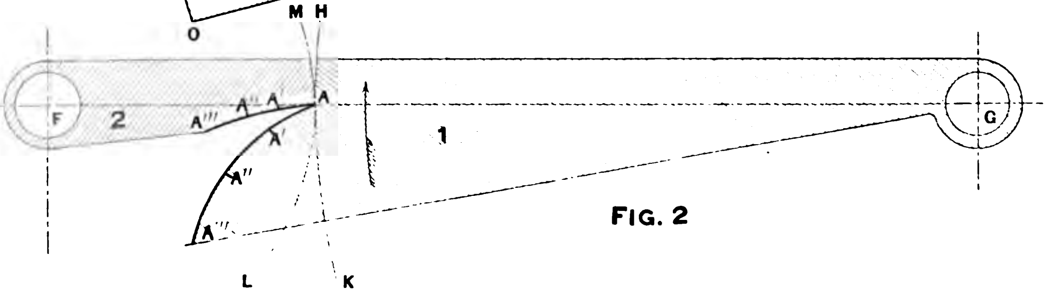


FIG. 2

PLATE 4.

Cycloidal Gear, Pinion and Rack having action on one side of pitch point. Limiting case.

REFERENCES TO TEXT.

Art.	Page	Art.	Page
15,	10.	24,	14.
16,	10.	25,	14.
18,	12.	26,	14.
19,	12.	36,	24.
20,	12.	47,	37.
23,	13.		

PLATE 4.

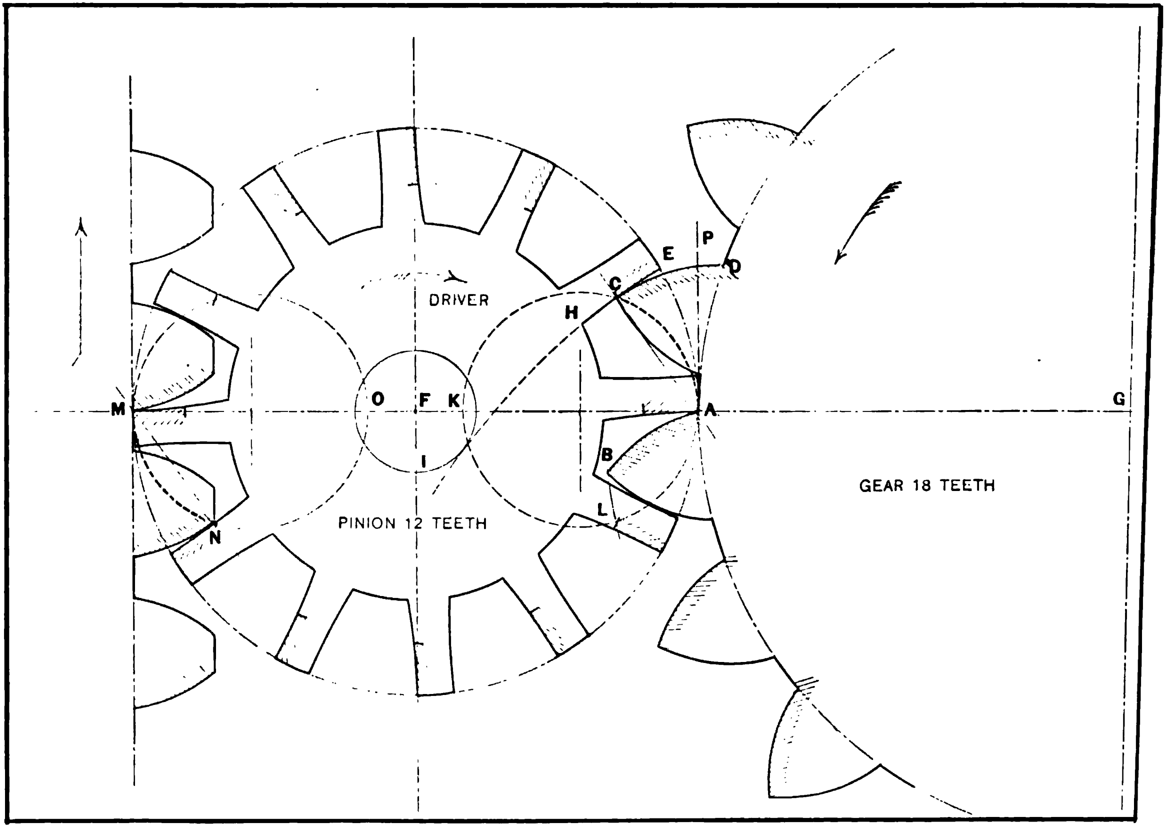


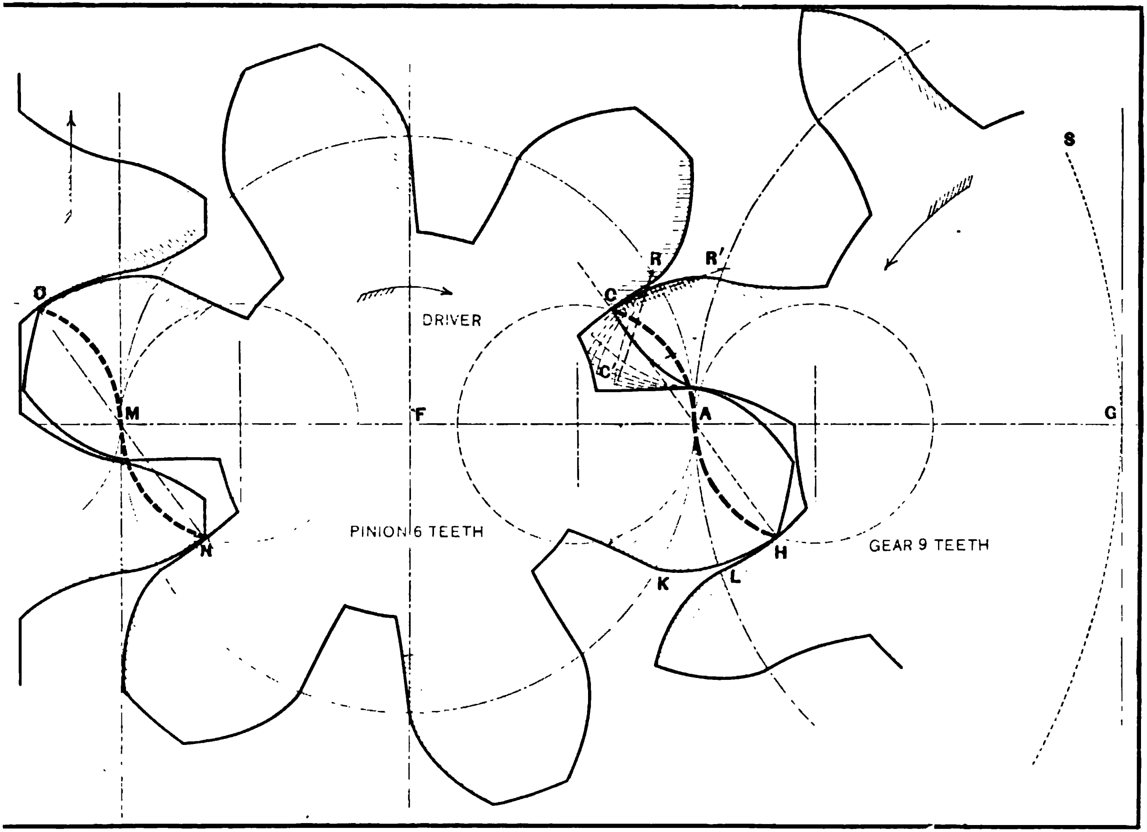
PLATE 5.

Cycloidal Gear, Pinion and Rack having action on both sides of the pitch point. Limiting case.

REFERENCES TO TEXT.

ART. 23,	Page 13.		ART.	36,	Page 24.
26,	14.			49,	38.
28,	15.		PROB.	2,	72.
32,	19.				

PLATE 5.



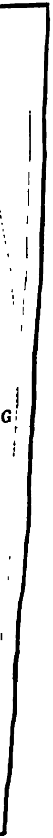


PLATE 6.

Cycloidal Gear, Pinion and Rack. Practical case.

REFERENCES TO TEXT.

ART.	34,	Page	21.
	36,		24.
	37,		25.

PLATE 6.

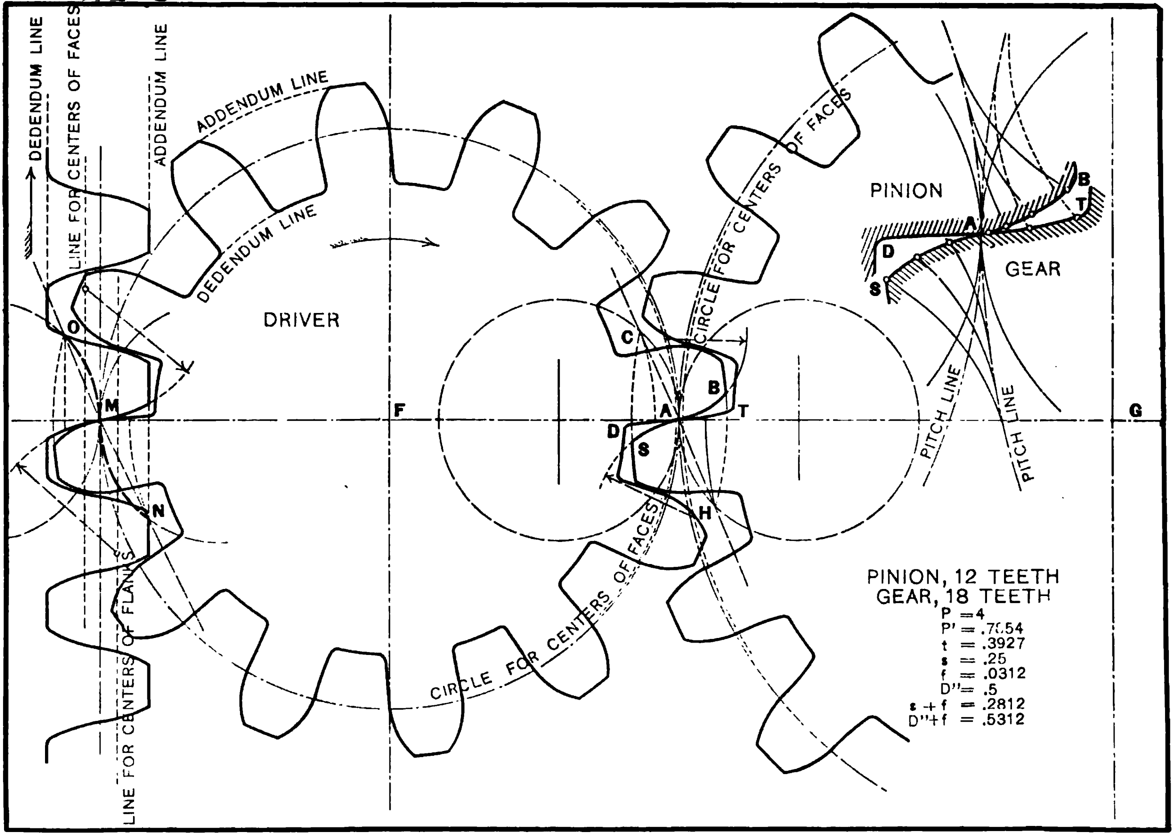

DEDENDUM LINE
LINE FOR CENTERS OF FACES
ADDENDUM LINE
ADDENDUM LINE
DEDENDUM LINE
DRIVER
O
M
N
F
LINE FOR CENTERS OF FLANKS
LINE FOR CENTERS OF FLANKS
CIRCLE FOR CENTERS OF FACES
CIRCLE FOR CENTERS OF FACES
C
B
A
T
D
S
H
PINION
GEAR
B
T
A
D
S
PITCH LINE
PITCH LINE
G
PINION, 12 TEETH
GEAR, 18 TEETH
P = 4
P' = .7854
t = .3927
s = .25
f = .0312
D'' = .5
s + f = .2812
D''+f = .5312

PLATE 7.

Involute Gear and Pinion. Limiting case. Mechanical method for describing the Involute.

REFERENCES TO TEXT.

Art. 38, Page 26.
39, 27.
42, 31.

PLATE 7.

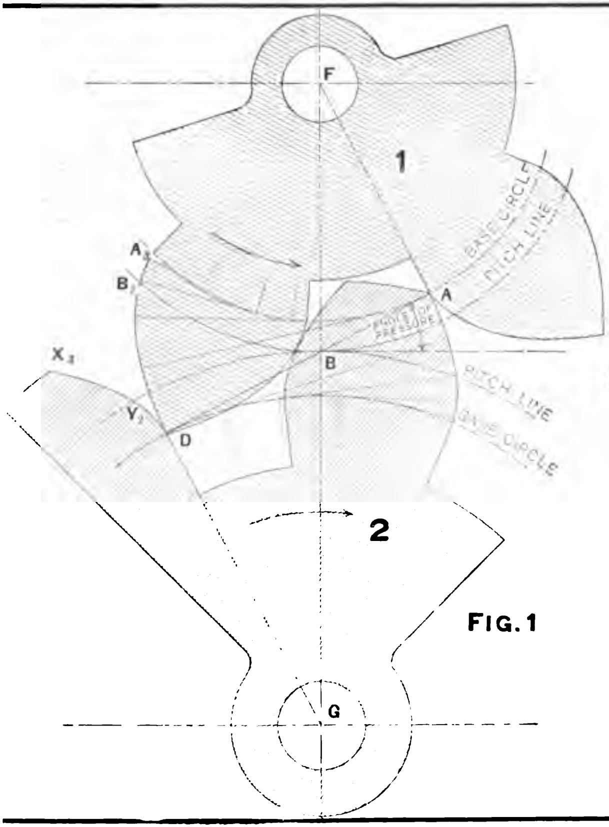


FIG. 1

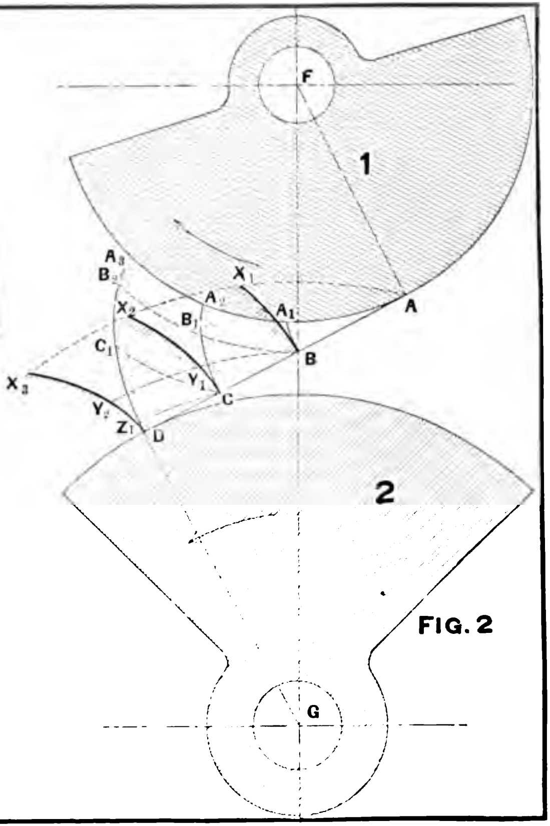


FIG. 2

PLATE 8.

Involute Gear, Pinion and Rack. Limiting case.

REFERENCES TO TEXT.

ART.	39,	Page	28.	ART.	41,	Page	30.
	40,		29.	PROB.	4,		74.

PLATE 8.

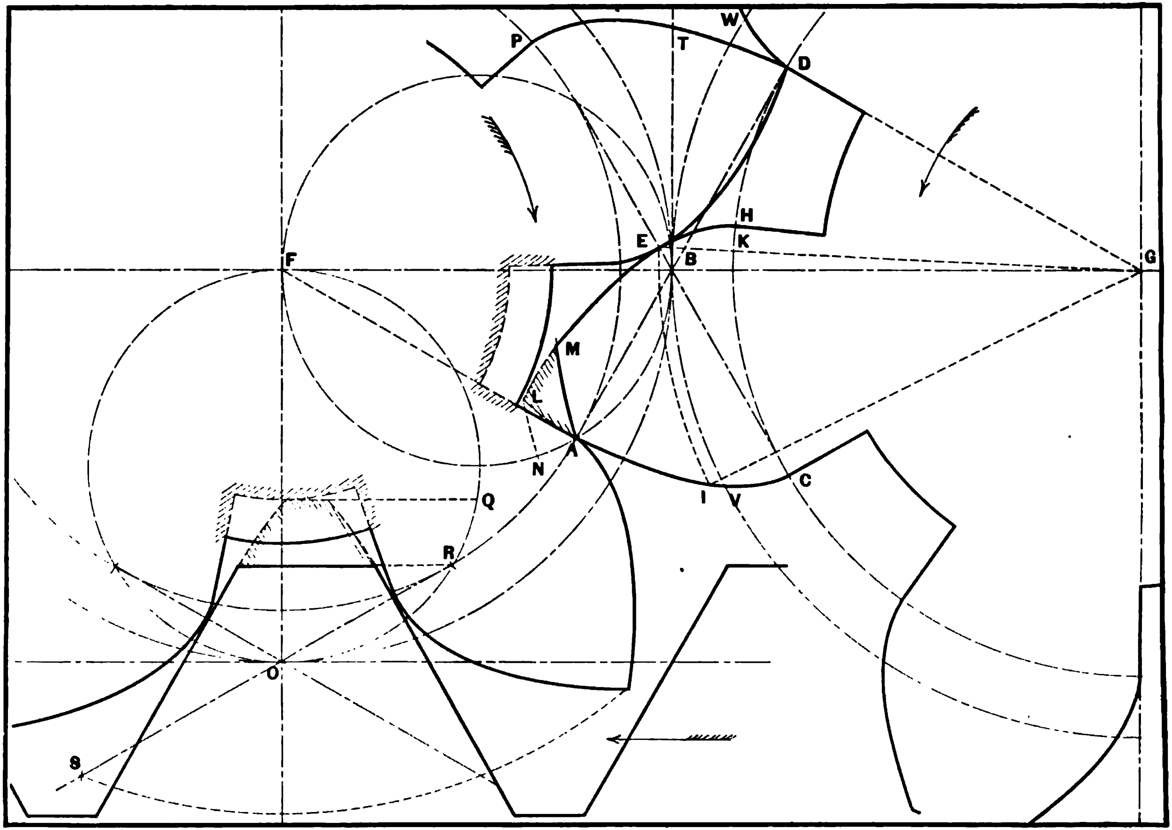


PLATE 9.

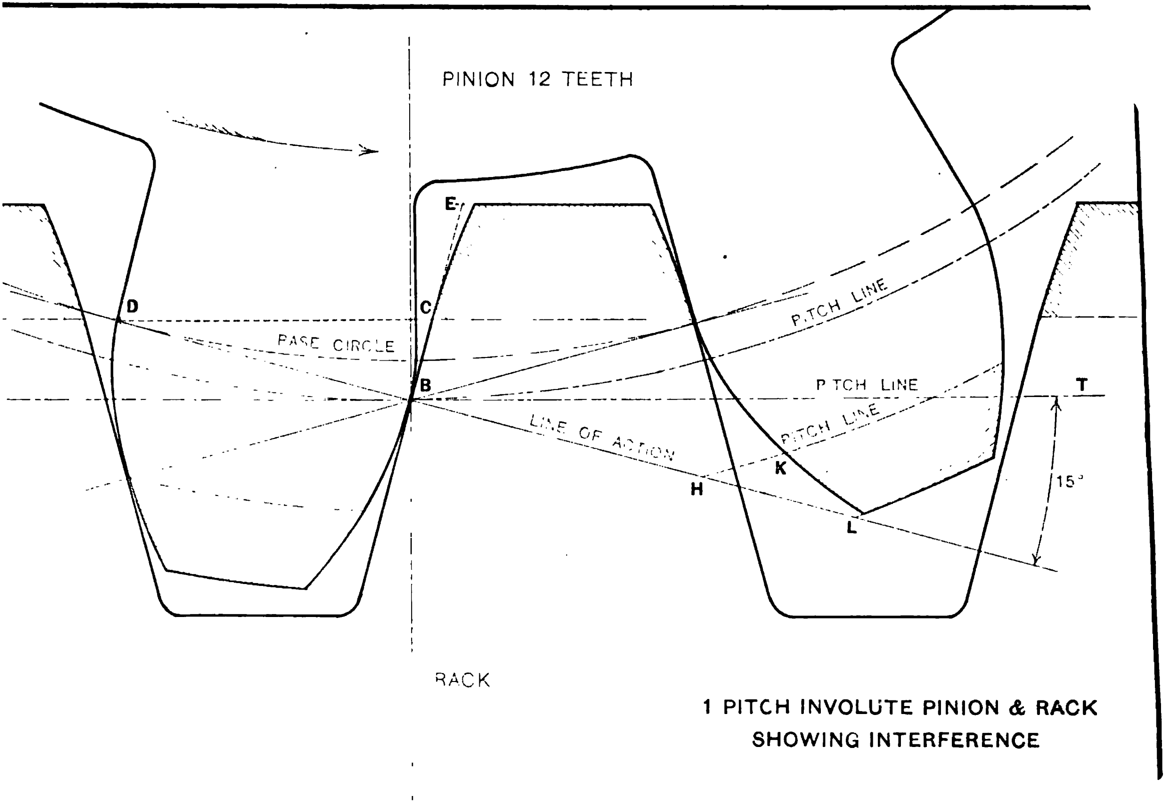
One Pitch Involute Gear and Pinion, showing Interference.

REFERENCES TO TEXT.

ART. 42, Page 30.
43, 32.

ART. 44, Page 33.
46, 35.

PLATE 10.



1 PITCH INVOLUTE PINION & RACK
SHOWING INTERFERENCE

PLATE II.

Annular Gearing.

REFERENCES TO TEXT.

ART. 49, Page 38.
50, 39.
ART. 52, Page 41.
54, 42.

P

PLATE 11

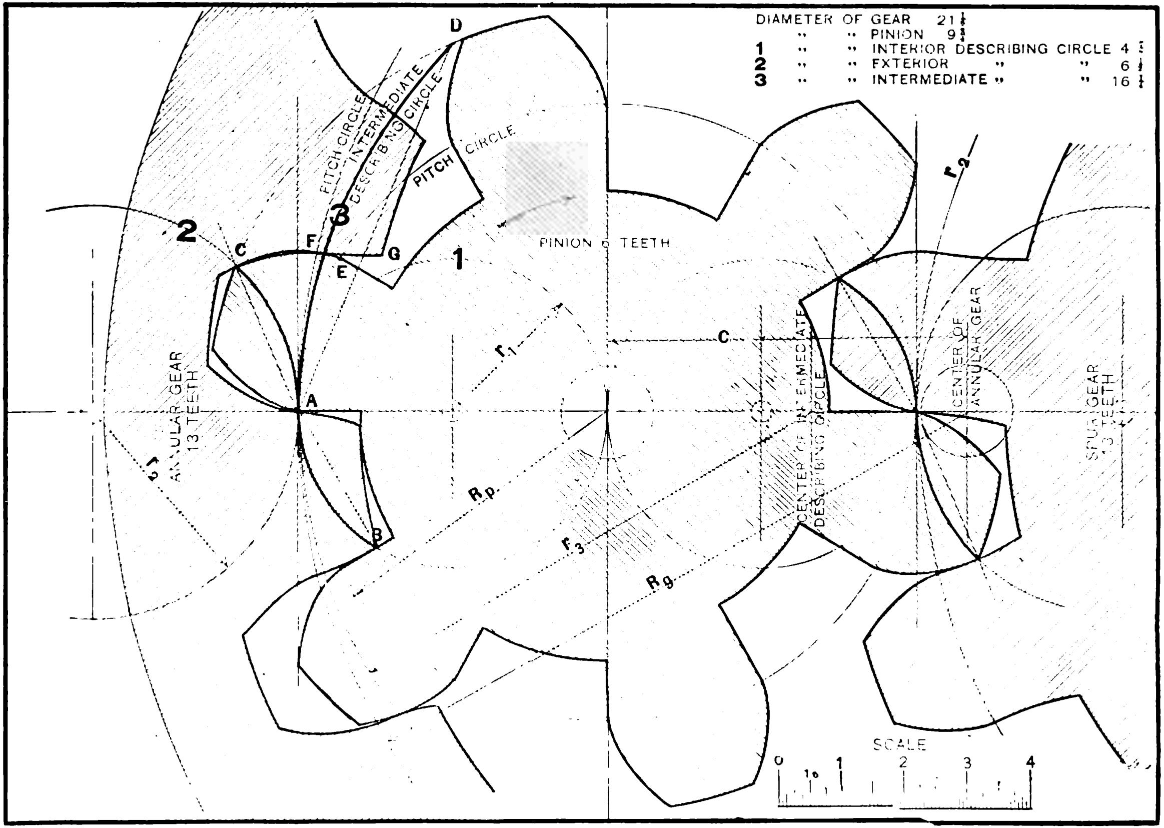

DIAMETER OF GEAR 21 1/8
" " PINION 9 3/4
1 " " INTERIOR DESCRIBING CIRCLE 4 5/8
2 " " FXTERIOR " " 6 1/2
3 " " INTERMEDIATE " " 16 1/4
D
FITCH CIRCLE
INTERMEDIATE
DESCR B NG CIRCLE
PITCH CIRCLE
3
2
C
F
E
G
1
PINION 6 TEETH
A
B
ANNULAR GEAR 13 TEETH
r_2
r_1
R_p
r_3
R_g
C
CENTER OF INTERMEDIATE DESCRIBING CIRCLE
CENTER OF ANNULAR GEAR
SPUR GEAR 13 TEETH
r_2
SCALE
0
1
2
3
4

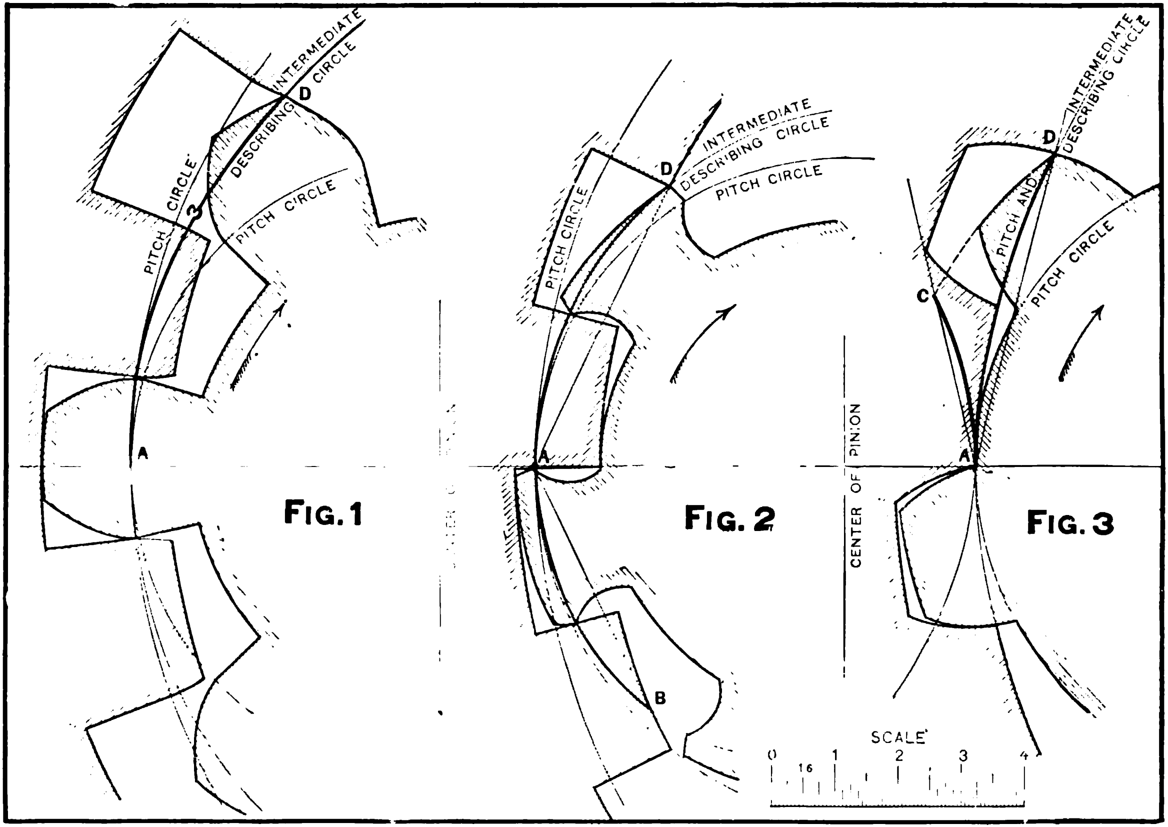
PLATE 12.

Annular Gearing. Special cases.

REFERENCES TO TEXT.

ART.	50,	Page	39.
	51,		40.
	52,		40.

PLATE 12



INTERMEDIATE DESCRIBING CIRCLE
D
PITCH CIRCLE
C
PITCH CIRCLE
A
FIG. 1
INTERMEDIATE DESCRIBING CIRCLE
D
PITCH CIRCLE
PITCH CIRCLE
A
B
FIG. 2
CENTER OF PINION
D
INTERMEDIATE DESCRIBING CIRCLE
PITCH AND PITCH CIRCLE
C
A
FIG. 3
SCALE
0
1/16
1
2
3
4

PLATE 13.

Bevel Gearing.

REFERENCES TO TEXT.

ART. 50, Page 47.   ART. 60, Page 48.

PLATE 13

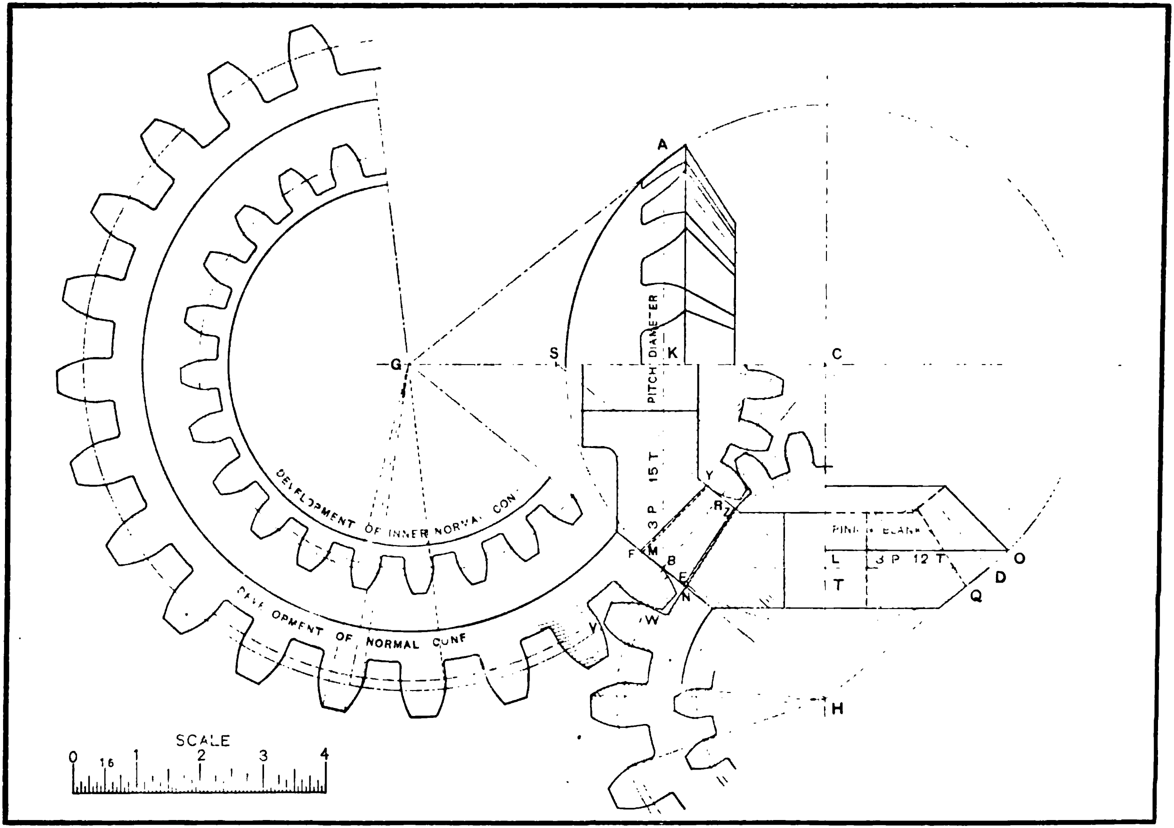

A
G
S
K
C
PITCH DIAMETER
3 P 15 T
Y
R
Z
F
M
B
E
N
W
Y
PINION BLANK
L
T
3 P 12 T
O
D
Q
H
DEVELOPMENT OF INNER NORMAL CONE
DEVELOPMENT OF NORMAL CONE
SCALE
0
1/16
1
2
3
4

PLATE 14.

Problems 1 to 4 inclusive.

REFERENCES TO TEXT.

ART.	65,	Page	69.	PROB.	3,	Page	72.
PROB.	1,		69.		4,		74.
	2,		71.				

PLATE 14

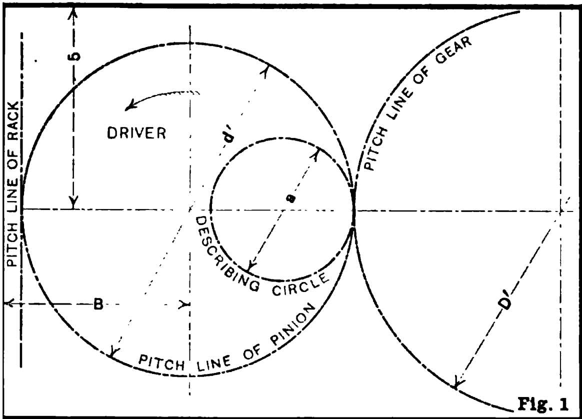


Fig. 1

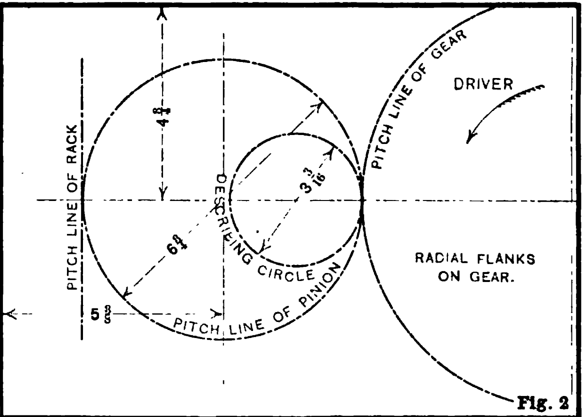


Fig. 2

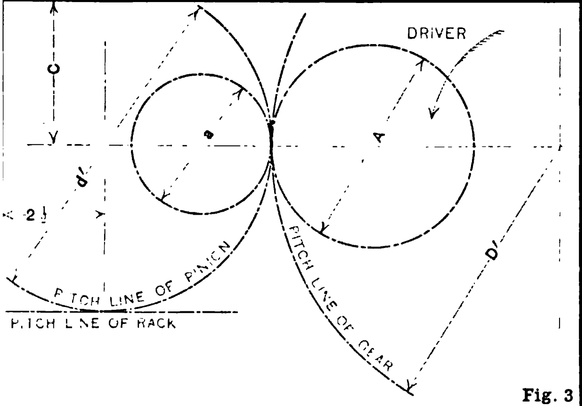


Fig. 3

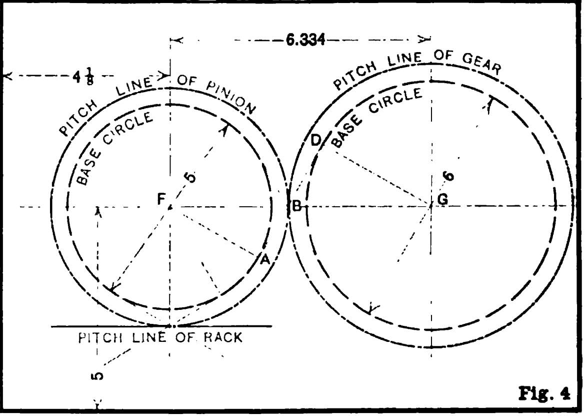


Fig. 4

PLATE 15.

Problems 5 to 9 inclusive.

REFERENCES TO TEXT.

Art.	65,	Page	69.	Prob.	7,	Page	77.
Prob.	5,		75.		8,		78.
	6,		76.		9,		79.

PLATE 15

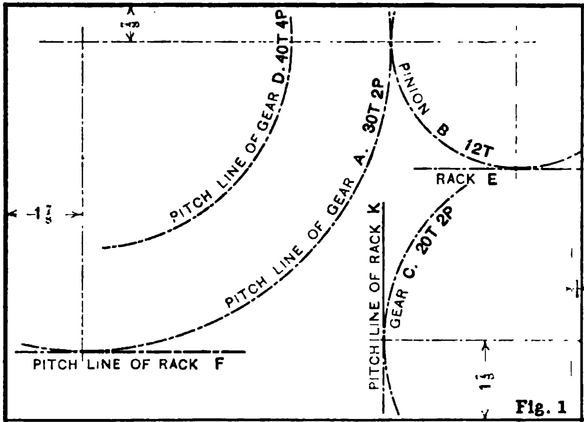


Fig. 1

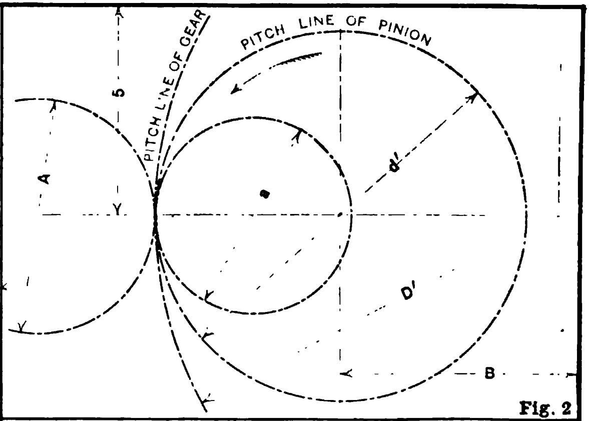


Fig. 2

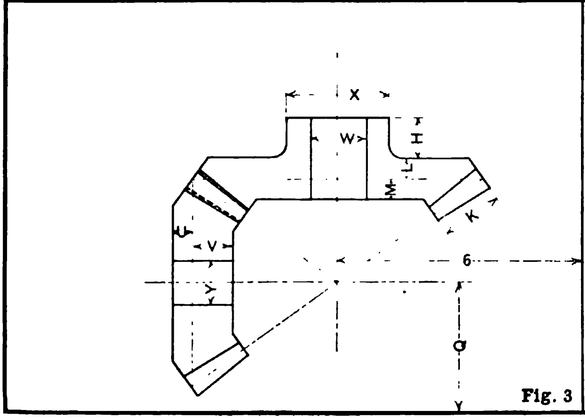


Fig. 3

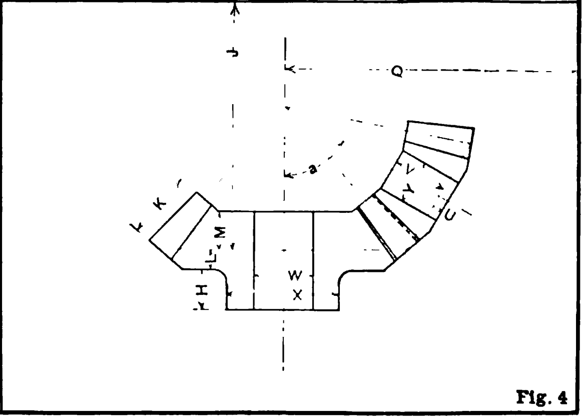


Fig. 4

29 11\05 31
47252

89088931688

b89088931688a

89088931688

B89088931688A